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Construction of $SU(N)$ Projection Operators: Applications to Particle and Nuclear Physics

By

M. Sc. Johann Edir Hernández Ybarra
Supervisor: Dr. Rubén Flores Mendieta

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Universidad Autónoma de San Luis Potosí, San Luis Potosí, México

Abstract

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M. Sc. Johann Edir Hernández Ybarra

PhD Thesis
Instituto de Física
Universidad Autónoma de San Luis Potosí

Abstract: Projection operators for irreducible representation of $SU(N)$ have been constructed, and a method to decompose any tensor operator which transforms under this group is described. Two applications for the method have been developed, the first one in the context of the $1/N_c$ expansion of QCD, and a second one for the interacting boson model (IBM).

Key words: projection operators, $SU(N)$ group, particle physics , nuclear physics, effective field theory.

Contents

Chapter 1

Introduction

The concept of symmetry is relevant in all areas of physics. Specifically, gauge symmetry is important for the description of elementary particles and the interactions into the *Standard Model* (SM) of particle physics. These symmetries have a well-defined mathematical structure in the scheme of Group Theory.

One of the earliest application of symmetry groups was the study of the spin of particles using representation theory and the $SU(2)$ group, that is the special unitary group of dimension three. In a similar way, the advances in nuclear physics, lead to describe the bond between protons and neutrons inside the nucleus via a strong force interaction, which has an $SU(2)$ invariance called isospin symmetry.

Then, in the decade of the 1960s, some different strong interacting particles were discovered, so Gell-Mann proposed an organizational method for hadrons using irreducible representations (irreps) of $SU(3)$. Since the method uses the eight-dimensional adjoint representation of $SU(3)$, it received the name of *eightfold way* [1].

Consequently, the description for hadrons given by Gell-Mann, led us to the discovery of three light quarks u, d and s, which fit the three dimensional representation of $SU(3)$. This symmetry has the name of $SU(3)$ flavor symmetry, and hadrons have a description as representation multiplets of this group. Moreover, the sector of SM related to the strong interaction is addressed by quantum chromodynamics (QCD), which is a gauge theory of quarks and gluons, where each quark comes in three identical states called colors. So the symmetry for QCD is $SU(3)$ color symmetry [2].

In general, the Standard Model describes three fundamental interactions through the local $SU(3) \otimes SU(2) \otimes U(1)$ gauge symmetry, where $SU(3)$ is related to the strong force and the product $SU(2) \otimes U(1)$ describes the electromagnetic and week interactions.

The examples above are theories described by the special unitary group, but there are many other examples. For instance, in order to understand the strongly coupled particles at low energies in QCD, where computing properties is complicated due to the absence of an small parameter, it is possible to consider a large number of colors. So, 't Hooft proposed that gauge theories based on the $SU(N_c)$ group simplify in the limit $N_c \to \infty$, where N_c is the number of colors [3], and Witten developed the first study of large-N_c for baryons [4]. Later, Dashen, Jenkins and Manohar showed that in the large-N_c limit, the baryon sector posses a contracted $SU(2) \otimes SU(N_f)$ spin-flavor symmetry, where N_f is the number of light quark flavors. Using the spin-flavor symmetry, it is possible to construct a $1/N_c$ expansion for any physical operator of QCD [5].

Additionally, the $SU(N)$ group is useful in the analysis of some models of atomic and nuclear physics. For example, the interacting boson model (IBM) possess a formulation based on dynamical symmetries considering different chains of groups. Different subsets of nuclei are described by different chains. In particular, the rotational subset of nuclei is described by $U(6) \supset SU(3) \supset SO(3)$, where $SU(3)$ group appears once again [6–8].

Since the special unitary group describes symmetries in some different theories in nuclear and particle physics. The aim of this work is to present a method to construct projection operators for $SU(N)$ in terms of the corresponding Casimir operators. The projection operators act on tensor operators that belong to tensor products of adjoint representation spaces, decomposing them into different operators which transform under a particular irreducible representation of the group [9].

Chapter 2

Projection Operators for Irreducible Representations of $SU(N)$

The $SU(N)$ group has many different applications in particle and nuclear physics, from its algebra to the representation theory of the group, it describes some relevant physical symmetries. However, since the theories that involve this symmetry contain physical quantities represented by operators that transform under particular representations of the group, it could be helpful to construct a method to identify the different contributions from all the representations included in those operators.

2.1 Lie groups

In order to define the $SU(N)$ group and describe its structure, it is necessary to define a group and some other mathematical objects. A **group** is a set of elements $G = \{a, b, c, ...\}$, which satisfy a multiplication rule ($\circ$) with the following properties:

- The product $a \circ b$ is an element of the group. It means that the group is closed under this operation.
- The set G contains a unit element e, which satisfies $a \circ e = e \circ a = a$ for an arbitrary element $a \in G$.
- For any element $a \in G$, there exists an inverse element a^{-1}, such that $a \circ a^{-1} = a^{-1} \circ a = e$.

- The multiplication is *associative*, i.e. $(a \circ b) \circ c = a \circ (b \circ c)$.

Another important concept is the **representation** of a group. A representation of G is a mapping, D of the elements of the group onto a set of linear operators with the properties:

- $D(e) = \mathbb{1}$, where $\mathbb{1}$ is the identity operator in the space on which linear operators act.
- $D(a)D(b) = D(ab)$, where $a, b \in G$, and it implies that the group multiplication rule is mapped onto the multiplication in the linear space on which linear operators act.

Also, a representation is *reducible* if it has an invariant subspace S, which implies that the action of the map D on any vector $s \in S$ is still in the subspace. Otherwise, if the representation does not have an invariant subspace, it is defined as an **irreducible representation (irrep)** [10].

Then, there are some properties that define certain types of groups. First, if group multiplication is commutative, the group is called *abelian*. If the elements of the group are functions of one or more continuous parameters, this is a *continuous* group, and if continuous variations of the parameters lead from any arbitrary element of the group to another, the group is called *continuously connected*. Finally, if for each sequence within the group, there exists an infinite partial sequence $\{a_n\}$ of group elements, and it converges to an element of the group, i.e. $\lim_{n\to\infty} a_n = a$, then the group is defined as a *compact* group [11].

In this way, a **Lie group** contains elements labeled by a set of continuous parameters with a multiplication rule that depends on the parameters themselves [10]. Additionally, a Lie group is compact if the parametrization consists of a finite number of bounded parameter domains.

The elements of a Lie group have the next structure

$$\exp\left[i\sum_a \beta^a X^a\right], \tag{2.1}$$

where β^a,$a = 1, ..., N$ are real numbers and X^a are linearly independent Hermitian operators[1]. The X^a are referred to as the generators of the Lie group and satisfy the commutation relations

$$\left[X^a, X^b\right] = i f^{abc} X^c. \tag{2.2}$$

[1] In this work, sum over repeated indices will follow the Einstein convention, i.e. $M^a N^a = \sum_a M^a N^a$.

The f^{abc} are called the structure constants of the group. The vector space $\beta^a X^a$, together with the commutation relations (2.2), defines the Lie algebra associated with the Lie group and is completely determined by the structure constants.

The generators of the group satisfy the Jacobi identity,

$$\left[X^a,\left[X^b,X^c\right]\right]+\left[X^b,\left[X^c,X^a\right]\right]+\left[X^c,\left[X^a,X^b\right]\right]=0, \tag{2.3}$$

and it could be also written in terms of the structure constants as

$$f^{bce}f^{aeg}+f^{abe}f^{ceg}+f^{cae}f^{beg}=0. \tag{2.4}$$

Also, there is an operator which commutes with the generators of the group, the **quadratic Casimir operator**, which is defined as

$$C\equiv X^eX^e. \tag{2.5}$$

so it follows that $[C,X^a]=0$.

Now, considering the structure constants, they can generate a representation of the algebra called the **adjoint** (or regular) representation. This is by defining a set of matrices T_A^a such that

$$[T_A^a]^{bc}\equiv -if^{abc}, \tag{2.6}$$

this constitutes a matrix representation, and its dimension is just the number of independent generators, which is the number of parameters required to describe a group element. Since the generators in the adjoint representation constitute a linear space, it is convenient to define a scalar product, to turn it into a vector space. A suitable product is the trace in the adjoint representation given by

$$\mathrm{Tr}(T_A^cT_A^d)=f^{abc}f^{abd}=N\delta^{cd}\equiv C_A\delta^{cd}, \tag{2.7}$$

which is a real symmetric matrix and C_A stands for the Casimir operator of the adjoint representation.

2.1.1 $SU(N)$ group

The special unitary group $SU(N)$ is generated by the Hermitian traceless $N \times N$ matrices. For this group, there are $N^2 - 1$ independent generators, so the dimension of the $SU(N)$ algebra is $N^2 - 1$. In the fundamental representation of the group, the normalization for the generators is given by the scalar product

$$\mathrm{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab}, \tag{2.8}$$

and the Casimir operator C_F for this representation is given as

$$\sum_a T^a_{AB} T^a_{BC} = \frac{N^2-1}{2N}\delta_{AC} \equiv C_F \delta_{AC}, \tag{2.9}$$

where T^a_{AB} is the matrix representation of the generators.

Considering the adjoint representation, $SU(N)$ arbitrary adjoint operators Q^a can be defined, such that

$$\left[T^a, Q^b\right] = i f^{abc} Q^c. \tag{2.10}$$

The operators Q^a can make up a basis for the vector space associated with the representation, also known as the *carrier space*, where the generators of the algebra of $SU(N)$ in the adjoint representation act. So, considering the adjoint representation of the generators T^a_A, the commutation relation above could be rewritten as

$$T^a_A Q^b = i f^{abc} Q^c. \tag{2.11}$$

Moreover, the carrier space generated by the operators Q^a is known as the *adjoint space*, and it is denoted by $adj = \{Q^a\}$. Then, it is possible to construct another tensor space, by taking products of adj with itself n times. This new space is denoted by $\prod_{i=1}^n$, and it can be decomposed into subspaces labeled by their eigenvalues of the quadratic Casimir operator. A simple way to perform the decomposition is by adapting the projector technique for reducible representations introduced in Ref. [9].

2.2 Projector Technique for $SU(N)$

Since **tensor operators** of $SU(N)$, which are sets of operators that transform under commutation with generators of a Lie algebra as irreps of the algebra, appear in some different physical theories, it is helpful to construct a method to categorize the contribution from each irrep to the operators.

From the work developed by Banda and Kirchbach [9], the decomposition of the space given by the product of adjoint representations, where the tensor operators act, can be achieved by adapting the projector technique for decomposing reducible representations. In this formalism, there are **projection operators** (projectors) are defined as

$$\mathcal{P}^{(m)} = \prod_{i=1}^{k} \left[\frac{C - c_n}{c_m - c_{n_i}} \right], \qquad c_m \neq c_{n_i}, \tag{2.12}$$

where k labels the number of different possible eigenvalues for the quadratic Casimir operator and c_m are its eigenvalues, which can be obtained from [12]:

$$c_m = \frac{1}{2} \left[nN - \frac{n^2}{N} + \sum_i r_i^2 + \sum_i c_i^2 \right] \tag{2.13}$$

where n is the total number of boxes of the Young tableau for a specific representation, r_i is the number of boxes in the ith row, and c_i is the number of boxes in the ith column.

Considering a $SU(N)$ tensor operator in the form $\prod_n^{i=1} Q_i^{a_i}$, where each $Q_i^{a_i}$ satisfies the commutation relation (2.10), the projectors act over the tensor operator as

$$\mathcal{P}^{(m)} \prod_{i=1}^{n} Q_i^{a_i} = \tilde{Q}^{a_1 \ldots a_n}, \tag{2.14}$$

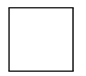
where the tensor $\tilde{Q}^{a_1 \ldots a_n}$ is an eigenstate for the quadratic Casimir C with eigenvalue c_m, such that

$$C\tilde{Q}^{a_1 \ldots a_n} = c_m \tilde{Q}^{a_1 \ldots a_n} \tag{2.15}$$

Thus, it is possible to obtain a particular component for any $SU(N)$ tensor operators, which transforms under a particular irreducible representation.

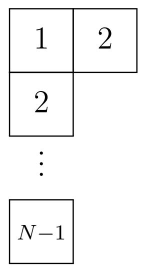
2.2.1 Projection Operators for Tensor Space $adj \otimes adj$

Any irreducible representation of $SU(N)$ that can be described by a multiplet has a representation using Young tableau [10]. For example, the fundamental representation is given by just one box as



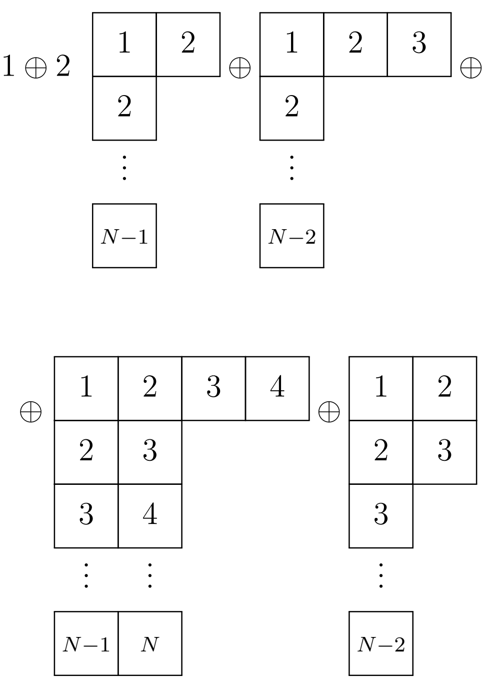
In this way, the adjoint representation can be written as

$adj =$



Following the product rules for Young tableau, the tensor product $adj \otimes adj$ has the decomposition

$adj \otimes adj = 1 \oplus 2$



Using the notation from [5], the representations from the expansion are

$$\bar{a}s = \begin{array}{ccc} \boxed{1} & \boxed{2} & \boxed{3} \\ \boxed{2} & & \\ \vdots & & \\ \boxed{N-2} & & \end{array} \qquad \bar{s}a = \begin{array}{ccc} \boxed{1} & \boxed{2} & \boxed{3} \\ \boxed{2} & \boxed{3} & \boxed{4} \\ \boxed{3} & \boxed{4} & \\ \vdots & \vdots & \\ \boxed{N-1} & \boxed{N} & \end{array}$$

$$\bar{s}s = \begin{array}{cccc} \boxed{1} & \boxed{2} & \boxed{3} & \boxed{4} \\ \boxed{2} & \boxed{3} & & \\ \boxed{3} & \boxed{4} & & \\ \vdots & \vdots & & \\ \boxed{N-1} & \boxed{N} & & \end{array} \qquad \bar{a}a = \begin{array}{cc} \boxed{1} & \boxed{2} \\ \boxed{2} & \boxed{3} \\ \boxed{3} & \\ \vdots & \\ \boxed{N-2} & \end{array},$$

and the complete decomposition of the product is

$$adj \otimes adj = 1 \oplus 2\, adj \oplus \bar{a}s \oplus \bar{s}a \oplus \bar{s}s \oplus \bar{a}a, \tag{2.16}$$

and the quadratic Casimir eigenvalues c_m for each representation are

$$\begin{gathered} 1 : \, c_0 = 0, \qquad adj : \, c_1 = N, \qquad \bar{a}s \oplus \bar{s}a : \, c_2 = 2N, \\ \bar{s}s : \, c_3 = 2(N+1), \qquad \bar{a}a : \, c_4 = 2(N-1). \end{gathered}$$

These eigenvalues can be obtained from the Young tableau related to each representation using the formula (2.13).

Taking the definition from (2.12), assuming that there are five representations with their respective eigenvalues, the projectors are given by

$$\mathcal{P}^{(m)} = \frac{\alpha_0 - \alpha_1 C + \alpha_2 C^2 - \alpha_3 C^3 + C^4}{\prod_{i=1}^{4} (c_m - c_{n_i})}, \qquad \text{with} \qquad c_m \neq c_{n_i}, \tag{2.17}$$

where the coefficients are given by combinations of products of the eigenvalues as

$$
\begin{aligned}
\alpha_0 &= c_{n_1}c_{n_2}c_{n_3}c_{n_4},\\
\alpha_1 &= c_{n_1}c_{n_2}c_{n_3} + c_{n_1}c_{n_2}c_{n_4} + c_{n_1}c_{n_3}c_{n_4} + c_{n_2}c_{n_3}c_{n_4},\\
\alpha_2 &= c_{n_1}c_{n_2} + c_{n_1}c_{n_3} + c_{n_1}c_{n_4} + c_{n_2}c_{n_3} + c_{n_2}c_{n_4} + c_{n_3}c_{n_4},\\
\text{and} \quad \alpha_3 &= c_{n_1} + c_{n_2} + c_{n_3} + c_{n_4}.
\end{aligned}
$$

As an important remark, notice that representations $\bar{a}s$ and $\bar{s}a$ are complex conjugated of each other, so they share the same eigenvalue. For this reason, it is necessary to construct a projection operator that includes both representations.

To compute the projectors, it is necessary to construct the quadratic Casimir for this space and it requires the generators T^a_{2A}. Since those generators act in the tensor space $adj \otimes adj$, it is possible to write them in terms of T^a_A as

$$T^a_{2A} = T^a_A \otimes \mathbb{1} + \mathbb{1} \otimes T^a_A, \tag{2.18}$$

so quadratic Casimir follows

$$
\begin{aligned}
C &= T^c_{2A}T^c_{2A}\\
&= \left(T^c_A \otimes \mathbb{1} + \mathbb{1} \otimes T^c_A\right)\left(T^c_A \otimes \mathbb{1} + \mathbb{1} \otimes T^c_A\right)\\
&= T^c_A T^c_A \otimes \mathbb{1} + \mathbb{1} \otimes T^c_A T^c_A + 2\, T^c_A \otimes T^c_A.
\end{aligned} \tag{2.19}
$$

By Schur's lemma, the quadratic Casimir operator for adj satisfies $T^c_A T^c_A = N\mathbb{1}$, then

$$C = 2\left(N\,\mathbb{1} \otimes \mathbb{1} + T^c_A \otimes T^c_A\right), \tag{2.20}$$

and its action over a tensor operator $Q^{b_1}_1 Q^{b_2}_2$ follows, from (2.11),

$$C Q^{b_1}_1 Q^{b_2}_2 = 2\left(N\delta^{b_1a_1}\delta^{b_2a_2} - f^{b_1a_1c}f^{b_2a_2c}\right) Q^{a_1}_1 Q^{a_2}_2. \tag{2.21}$$

Hence, in components, C reads

$$\begin{aligned}[C]^{a_1a_2b_1b_2} &= 2\left(N\,\delta^{b_1a_1}\delta^{b_2a_2} - F^{b_1a_1b_2a_2}\right) \\ &= \frac{4}{N}\left[\frac{N^2}{2}\delta^{b_1a_1}\delta^{b_2a_2} - \delta^{b_1b_2}\delta^{a_1a_2} + \delta^{b_1a_2}\delta^{b_2a_1}\right] - 2\left(D^{b_1b_2a_1a_2} - D^{b_1a_2b_2a_1}\right),\end{aligned} \tag{2.22}$$

the second line follows the identities from Appendix A, and the tensors F and D are defined as

$$F^{a_1a_1b_1b_2} = f^{a_1a_2c}f^{b_1b_1c}, \qquad D^{a_1a_2b_1b_2} = d^{a_1a_2c}d^{b_1b_2c}, \tag{2.23}$$

and d^{abc} represents the fully symmetric coefficients, given by

$$d^{abc} = \frac{1}{4}\mathrm{Tr}\left(\left\{T^a, T^b\right\}T^c\right), \tag{2.24}$$

and consequently, it also appears in the anti-commutation relation of the generators T^a of $SU(N)$

$$\left\{T^a, T^b\right\} = \frac{1}{N}\delta^{ab}\mathbb{1} + d^{abc}T^c. \tag{2.25}$$

Since projector operators include powers of quadratic Casimir, these powers are computed by contracting the indices in the following way. For the second power

$$\left[C^2\right]^{a_1a_2b_1b_2} = [C]^{a_1a_2c_1c_2}\,[C]^{c_1c_2b_1b_2}, \tag{2.26}$$

for the third power

$$\left[C^3\right]^{a_1a_2b_1b_2} = \left[C^2\right]^{a_1a_2c_1c_2}\,[C]^{c_1c_2b_1b_2}, \tag{2.27}$$

and so on. Thus, from equation (2.17), the projectors are

$$\left[\mathcal{P}^{(0)}\right]^{a_1a_2b_1b_2} = \frac{1}{N^2-1}\delta^{a_1a_2}\delta^{b_1b_2}, \tag{2.28}$$

$$\left[\mathcal{P}^{(1)}\right]^{a_1a_2b_1b_2} = \frac{N}{N^2-4}D^{a_1a_2b_1b_2} + \frac{1}{N}F^{a_1a_2b_1b_2}, \tag{2.29}$$

$$\left[\mathcal{P}^{(2)}\right]^{a_1a_2b_1b_2} = \frac{1}{2}\left(\delta^{a_1b_1}\delta^{a_2b_2} - \delta^{a_2b_1}\delta^{a_1b_2}\right) - \frac{1}{N}F^{a_1a_2b_1b_2}, \tag{2.30}$$

$$\begin{aligned}\left[\mathcal{P}^{(3)}\right]^{a_1a_2b_1b_2} =& \frac{N+2}{4N}\left(\delta^{a_1b_1}\delta^{a_2b_2}+\delta^{a_2b_1}\delta^{a_1b_2}\right)-\frac{N+2}{2N(N+1)}\delta^{a_1a_2}\delta^{b_1b_2}\\ &-\frac{N+4}{4(N+2)}D^{a_1a_2b_1b_2}+\frac{1}{4}\left(D^{a_1b_1a_2b_2}+D^{a_2b_1a_1b_2}\right),\end{aligned} \tag{2.31}$$

$$\begin{aligned}\left[\mathcal{P}^{(4)}\right]^{a_1a_2b_1b_2} =& \frac{N-2}{4N}\left(\delta^{a_1b_1}\delta^{a_2b_2}+\delta^{a_2b_1}\delta^{a_1b_2}\right)+\frac{N-2}{2N(N-1)}\delta^{a_1a_2}\delta^{b_1b_2}\\ &+\frac{N-4}{4(N-2)}D^{a_1a_2b_1b_2}-\frac{1}{4}\left(D^{a_1b_1a_2b_2}+D^{a_2b_1a_1b_2}\right).\end{aligned} \tag{2.32}$$

The above operators satisfy two important properties :

- The projection operators are orthogonal by definition, so

$$\left[\mathcal{P}^{(m)}\right]^{a_1a_2c_1c_2}\left[\mathcal{P}^{(n)}\right]^{c_1c_2b_1b_2} = \begin{cases}0, & m\neq n\\ \left[\mathcal{P}^{(m)}\right]^{a_1a_2b_1b_2}, & m=n.\end{cases} \tag{2.33}$$

- The set of projection operators is complete, in the sense that the sum of all projectors in the set is equal to the identity

$$\sum_{m=0}^{4}\left[\mathcal{P}^{(m)}\right]^{a_1a_2b_1b_2} = \delta^{a_1b_1}\delta^{a_2b_2}. \tag{2.34}$$

Now, given two arbitrary adjoint operators $Q_1^{b_1}$ and $Q_2^{b_2}$, the action of projection operators $\mathcal{P}^{(m)}$ on the tensor operator $Q_1^{b_1}Q_2^{b_2}$ follows

$$\left[Q^{(0)}\right]^{b_1b_2} = \left[\mathcal{P}^{(0)}Q_1Q_2\right]^{b_1b_2} = \frac{1}{N^2-1}\delta^{b_1b_2}Q_1^eQ_2^e, \tag{2.35}$$

$$\begin{aligned}\left[Q^{(1)}\right]^{b_1b_2} &= \left[\mathcal{P}^{(1)}Q_1Q_2\right]^{b_1b_2}\\ &= \frac{N}{N^2-4}D^{b_1b_2a_1a_2}Q_1^{a_1}Q_2^{a_2}+\frac{1}{N}F^{b_1b_2a_1a_2}Q_1^{a_1}Q_2^{a_2},\end{aligned} \tag{2.36}$$

$$\begin{aligned}\left[Q^{(2)}\right]^{b_1b_2} &= \left[\mathcal{P}^{(2)}Q_1Q_2\right]^{b_1b_2}\\ &= \frac{1}{2}\left(Q_1^{b_1}Q_2^{b_2}-Q_1^{b_1}Q_2^{b_2}\right)-\frac{1}{N}F^{b_1b_2a_1a_2}Q_1^{a_1}Q_2^{a_2},\end{aligned} \tag{2.37}$$

$$\begin{aligned}
\left[Q^{(3)}\right]^{b_1b_2} &= \left[\mathcal{P}^{(3)}Q_1Q_2\right]^{b_1b_2} \\
&= \frac{N+2}{4N}\left(Q_1^{b_1}Q_2^{b_2}+Q_1^{b_2}Q_2^{b_1}\right)-\frac{N-2}{2N(N-1)}\delta^{b_1b_2}Q_1^eQ_2^e \\
&\quad+\frac{N+4}{4(N+2)}D^{b_1b_2a_1a_2}Q_1^{a_1}Q_2^{a_2}+\frac{1}{4}\left(D^{b_1a_1b_2a_2}+D^{b_1a_2b_2a_1}\right)Q_1^{a_1}Q_2^{a_2},
\end{aligned} \tag{2.38}$$

$$\begin{aligned}
\left[Q^{(4)}\right]^{b_1b_2} &= \left[\mathcal{P}^{(4)}Q_1Q_2\right]^{b_1b_2} \\
&= \frac{N-2}{4N}\left(Q_1^{b_1}Q_2^{b_2}+Q_1^{b_2}Q_2^{b_1}\right)+\frac{N-2}{2N(N-1)}\delta^{b_1b_2}Q_1^eQ_2^e \\
&\quad+\frac{N-4}{4(N-2)}D^{b_1b_2a_1a_2}Q_1^{a_1}Q_2^{a_2}-\frac{1}{4}\left(D^{b_1a_1b_2a_2}+D^{b_1a_2b_2a_1}\right)Q_1^{a_1}Q_2^{a_2}.
\end{aligned} \tag{2.39}$$

The operators above on the left-hand side are labeled by the space representation they belong to, since operators $P^{(m)}$ projectors out those particular components of each representation, respectively.

2.2.1.1 Projection Operators considering $N = 2$

The simplest case to apply the projector technique is considering the non-Abelian Lie group $SU(2)$. This group appears in the description of spin and isotopic spin (isospin) symmetries. In those cases, the generators of the group are J^i and I^a, which correspond to spin and isospin, respectively. Also, the structure constants for the commutation relations are ϵ^{ijk} $(i, j, k = 1, 2, 3)$ and $\epsilon^{abc}(a, b, c = 1, 2, 3)$, which are totally antisymmetric.

For this case, there are some important considerations. First, $SU(2)$ doesn't admit representations for the eigenvalues $c_2 = 2N$ and $c_4 = 2(N-1)$ of the quadratic Casimir operator in the $adj \otimes adj$. Then, the projector $\mathcal{P}^{(1)}$ is not well-defined in this case, so it takes a different form for $N = 2$. Thus, it is possible to construct the projectors associated with the eigenvalues c_0, c_1 and c_3 only.

While $\mathcal{P}^{(0)}$ is easily obtained from (2.28) as

$$\left[\mathcal{P}^{(0)}\right]^{a_1a_2b_1b_2} = \frac{1}{3}\delta^{a_1a_2}\delta^{b_1b_2}, \tag{2.40}$$

$\mathcal{P}^{(1)}$ considers the definition given by (2.12), so

$$\mathcal{P}^{(1)} = \frac{C^2-(c_0+c_3)\,C+c_0c_3}{(c_1-c_0)\,(c_1-c_2)}. \tag{2.41}$$

Considering the eigenvalues for $N = 2$, it follows that

$$\mathcal{P}^{(1)} = \frac{1}{8}\left(6C - C^2\right), \tag{2.42}$$

and using the expressions for C and C^2 from eqs. (2.22) and (2.26)

$$\left[\mathcal{P}^{(1)}\right]^{a_1a_2b_1b_2} = \frac{1}{2}\left(\delta^{a_1b_1}\delta^{a_2b_2} - \delta^{a_2b_1}\delta^{a_1b_2}\right). \tag{2.43}$$

Analogously,

$$\left[\mathcal{P}^{(3)}\right]^{a_1a_2b_1b_2} = \frac{1}{2}\left(\delta^{a_1b_1}\delta^{a_2b_2} + \delta^{a_2b_1}\delta^{a_1b_2}\right) - \frac{1}{3}\delta^{a_1a_2}\delta^{b_1b_2}. \tag{2.44}$$

The action of the above protectors over an arbitrary tensor operator $Q_1^{b_1}Q_2^{b_2}$ defined in spin space, project out the $J = 0, J = 1$, and $J = 2$ spin components of that tensor product, respectively. A similar result can be obtained for isospin space.

As remark. according to the projectors structure described above, a spin-0 operator can be obtained by contracting the spin indices of both operators Q_1 and Q_2. A spin-1 operator can be obtained from the antisymmetrization of the operators. Lastly, a spin-2 is given as the symmetrization of Q_1 and Q_2 subtracting the spin-0 component.

Chapter 3

$1/N_c$ Expansion for QCD

In particle physics, the Standard Model (SM) describes the four fundamental interactions of nature considering twelve fundamental particles (six quarks and six leptons) and three gauge bosons (gluon, W, Z). Quarks and gluons interact via the strong interaction, which confines quarks in hadron particles.

Quantum Chromodynamics (QCD) is the theory that describes the strong interactions in the Standard Model, and its development started since the proposition of the *Eightfold Way* by Gell-Mann [13]. Fundamentally, QCD is a theory of quarks and gluons, but at low energies, those particles are confined to baryons and mesons. Thus, there are some different approaches to aboard the low energy sector, where effective field theories stand out.

3.1 Quantum Chromodynamics

Quantum Chromodynamics is a non-Abelian gauge theory with gauge group $SU_c(3)$, where c stands for color, coupled to the fermions of the theory (quarks) in the fundamental representation. The most general $SU(N)$-invariant Lagrangian for a set of N fermions and N scalar interacting with non-Abelian gauge fields is

$$\begin{aligned}\mathcal{L} =& -\frac{1}{4}\left(F^a_{\mu\nu}\right)^2 - \frac{1}{2\xi}\left(\partial_\mu \bar{c}^a\right)\left(\delta_{ac}\partial_\mu + g f^{abc} A^b_\mu\right) c^c \\ &+ \bar{\psi}_i(\delta_{ij} i\partial\!\!\!/ + g A\!\!\!/^a T^a_{ij} - m\delta_{ij})\psi_j \\ &+ \left[\left(\delta_{ki}\delta_\mu - igA^a_\mu T^a_{ki}\right)\phi_i\right]^\star \left[\left(\delta_{kj}\delta_\mu - igA^a_\mu T^a_{kj}\right)\phi_j\right] - M^2 \phi^\star_i \phi_i,\end{aligned} \tag{3.1}$$

where ψ_i, ϕ_i and A^a_μ are fermion, scalar and gauge fields respectively, c^a and $\bar{c}^a$ are Feddeev-Popov ghost and anti-ghosts, and

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu. \tag{3.2}$$

This Lagrangian contains scalar fields even though there are no observed scalar states in nature that are colored. Nevertheless, some theories include them, for example, supersymmetric QCD [14].

From the Lagrangian, it is possible to obtain the Feynman rules and use them to obtain tree-level and 1-loop results. Those computations give us a result that QCD gauge coupling gets larger at larger distances, which is the opposite behavior from Quantum Electrodynamics (QED). This makes the phenomenology of QCD completely different from QED and it gives us two characteristic behaviors: *Asymptotic freedom* and *confinement*.

Confinement implies that quarks are strongly coupled at low energies in hadrons, and additionally, there is no expansion parameter to compute low energy properties of hadrons in QCD. However, t'Hoof noticed that QCD contains the parameter N_c (number of colors), and the theory simplifies in the limit $N_c \to \infty$, which is known as the **Large N_c limit** [3].

Even though this limit was proposed as a pure quantitative computational method, it has given qualitative predictions. For example, the properties of baryons have been studied in a systematic expansion in $1/N_c$, and also combined with some effective field theories [15–18].

3.2 Spin-flavor Algebra in Large N_c QCD

Considering the large N_c limit, the baryon sector of QCD possesses an exact contracted symmetry $SU(2N_f) = SU(2) \otimes SU(N_f)$, also known as the **spin-flavor symmetry**, where N_f is the number of light quark flavors. For this symmetry, the quark representation is based on the non-relativistic quark picture model.

In quark representation, there is a set of quark creation and annihilation operators $q^\dagger_\alpha$ and q^α, where the label $\alpha = 1, ..., N_f$ represents the spin-up quarks and $\alpha = N_f, ..., 2N_f$, the spin-down quarks. Then, from the antisymmetry of the $SU(N_c)$ color group and Fermi statistics, the ground state baryons contain N_c quarks in the completely symmetric representation of the spin-flavor group. This implies that it is possible to omit the color quantum numbers of quark operators and consider them as bosonic objects in the spin-flavor analysis [5].

Therefore, the quark operators satisfy the bosonic commutation relation

$$\left[q^{\alpha}, q^{\dagger}_{\beta}\right] = \delta^{\alpha}_{\beta}. \tag{3.3}$$

Quark operators can be classified according to 0-body, 1-body, 2-body,..., or n-body, operators, it depends on the number of q and $q^{\dagger}$ pairs that the operator contains. So, there is a unique zero-body operator, the identity operator $\mathbb{1}$, because it contains no q or $q^{\dagger}$. 1-body operators are the quark number operator $q^{\dagger}q$ and the *spin-flavor adjoint* $q^{\dagger}\Lambda^{A}q$, where $A = 1, ..., (2N_f)^2 - 1$, and Λ^A is a *spin-flavor generator*. 2-body operators involve two pairs q and $q^{\dagger}$, and so on. In general, a n- body operator is given as a combination of 1-body operators as polynomials of order n.

The 1-body operators $q^{\dagger}\Lambda^{A}q$ can be written into a representation of the decomposition of the spin-flavor group $SU(2N_f) = SU(2) \times SU(N_f)$ as:

$$\begin{aligned} J^i &= q^{\dagger}\left(J^i \otimes \mathbb{1}\right) q \qquad (1,0), \\ T^a &= q^{\dagger}\left(\mathbb{1} \otimes T^a\right) q \qquad (0,\mathrm{adj}), \\ G^i &= q^{\dagger}\left(J^i \otimes T^a\right) q \qquad (1,\mathrm{adj}), \end{aligned}$$

where J^i are the spin generators, T^a are the flavor generators, G^{ia} are the spin-flavor generators, and those generators satisfy the $SU(2N_f)$ commutation relations given by the *spin-flavor algebra* on Table 3.1.

$SU(2N_f)$ commutation relations
$[J^i, T^a] = 0$
$[J^i, J^j] = i\epsilon^{ijk}J^k, \quad \left[T^a, T^b\right] = if^{abc}T^c$
$[J^i, G^{ja}] = i\epsilon^{ijk}G^{ka}, \quad \left[T^a, G^{ib}\right] = if^{abc}G^{ic}$
$\left[G^{ia}, G^{jb}\right] = \frac{i}{4}\delta^{ij}f^{abc}T^c + \frac{i}{2N_f}\delta^{ab}\epsilon^{ijk}J^k + \frac{i}{2}\epsilon^{ijk}d^{abc}G^{kc}$

Table 3.1: Commutation relations for spin-flavor 1 -body operators.

All operators above transform under the representations of the $SU(2) \times SU(N_f)$ group given on the right of each one, and "adj" is the adjoint representation of $SU(N_f)$. The set of indices $(A, B, ...)$ indicates that the object transforms according to the adjoint representation of the $SU(2N_f)$ spin-flavor group, lowercase letters $(a, b, ...)$ denote indices that transform under the adjoint representation of the $SU(N_f)$ flavor group, and the set $(i, j...)$ is related to objects that transform as the vector representation of spin.

3.2.1 Expansion for QCD operators

In QCD, the baryons are color singlet states of N_c quarks [3], and considering the large N_c limit, any QCD operator has an expansion in $1/N_c$ in terms of operators in the quark representation. This expansion, at leading order, is

$$\mathcal{O}_{QCD} = \sum_{n,k} c_k^{(n)} \frac{1}{N_c^{n-1}} \mathcal{O}_k^{(n)}, \tag{3.4}$$

where the sum is over all the linearly independent n-body operators $\mathcal{O}_k^{(n)}$, with $n = 0, ..., N_c$ because there are N_c quarks in the baryons, and all $\mathcal{O}_k^{(n)}$, consider the same spin and flavor quantum numbers as $\mathcal{O}_{QCD}$, which implies that they transform under the same irreducible representation of the spin-flavor group. QCD dynamics is parametrized by the coefficients $c_k^{(n)}$, which are of order unity, but it is possible to compute subleading $1/N_c$ corrections to the leading order expansion by adding $1/N_c$ corrections to coefficients.

From the spin-flavor algebra, the commutator of an m-body with an n-body operator is an $(m + n - 1)$-body operator:

$$\left[\mathcal{O}^{(m)}, \mathcal{O}^{(n)}\right] = \mathcal{O}^{(m+n-1)}, \tag{3.5}$$

this relation is relevant for the classification and reductions of operators in computations of baryon properties and interactions. In contrast, the anticommutator does not reduce the counting on the operators, so $\left\{\mathcal{O}^{(m)}, \mathcal{O}^{(n)}\right\}$ is a $(m + n)$-body operator [5].

As an important remark, it is possible to construct a linearly independent and complete operator basis of n-body operators, with particular transformation properties, using the spin-flavor algebra, identities between n-body operators, identities for quarks operators, and the evaluation of matrix elements for the operators. Also, the operator basis for any $SU(2) \times SU(N_f)$ group representation contains a finite number of operators. However, the process of construction of a complete linearly independent basis of n-body operators for any

n is not well defined, so there is no systematic way to create it.

3.3 $1/N_c$ Expansion for Chiral Perturbation theory

The $1/N_c$ expansion provides us with a systematic description of QCD operators in terms of the spin-flavor algebra generators. However, it is not possible to describe the dynamics of the theory because there is no way to construct a Lagrangian at this level. In order to obtain a Lagrangian for baryons, some effective field theories are useful. Particularly, *Chiral Perturbation Theory* (ChPT) describes the strong interactions at low energies, considering a theory of pions and baryons instead of quarks and gluons.

The development of ChPT was started by Weinberg with his proposition of phenomenological Lagrangians to describe the low energy regime of QCD, for example, the computation soft pions matrix elements [19]. Chiral perturbation theory considers the light sector of quarks u, d and s, in the limit where their masses vanish, that transform under the *chiral symmetry group* $SU(3)_L \times SU(3)_R \times U(1)_V$ for the QCD Lagrangian. This symmetry is spontaneously broken to the group $SU(3) \times U(1)_V$ by QCD vacuum, and as a consequence, there is an octet of pseudoscalar Goldstone bosons, the pions [20]. The perturbative expansion of the theory uses pion momenta, quark masses m_q and scale parameter of chiral symmetry breaking $\Lambda_\chi \sim 1\text{GeV}$ as expansion parameters.

For baryons, there are some issues in the computation considering chiral symmetry, but these problems can be avoided considering baryons as heavy static fermions. The justification for this assumption is that the momentum transferred by pion exchange between baryons is small compared to the baryon mass [21].

An interesting approach is to combine the formalisms for the $1/N_c$ expansion and chiral perturbation theory for heavy baryons in just one method. This combined method constrains the low-energy interactions of hadrons with pion nonet π, K, η, and η' in a more effective theory. The expansion in mixed formalism considers a combined expansion in powers of m_q/Λ_χ and $1/N_c$ simultaneously about the double limit $m_q \to 0$ and $N_c \to \infty$ [22].

The $1/N_c$ expansion with chiral perturbation theory provides us with a $1/N_c$ chiral Lagrangian, which describes the interactions of baryons and low-momentum pions as

$$\mathcal{L} = \mathcal{L}_{\text{pion}} + \mathcal{L}_{\text{baryon}}, \tag{3.6}$$

where the pion contribution describes the self-interactions of the pseudo-Goldstone bosons, and the baryon contribution treats baryons as heavy static fields with fixed velocity v^μ. Since baryons have masses of order N_c and, in the large N_c limit, they become very heavy relative to mesons with masses of order 1, some pion contributions can be neglected at leading order in the Lagrangian.

The $1/N_c$ baryon chiral Lagrangian for arbitrary N_c can be written in the most general form as

$$\mathcal{L}_{\text{baryon}} = i\mathcal{D}^0 - \mathcal{M}_{\text{hyperfine}} + \text{Tr}\left(\mathcal{A}^k\lambda^c\right) A^{kc} + \frac{1}{N_c}\text{Tr}\left(\mathcal{A}^k \frac{2I}{\sqrt{6}}\right) A^k + ..., \tag{3.7}$$

where

$$\mathcal{D}^0 = \partial^0 \mathbb{1} + \text{Tr}\left(\mathcal{V}^0\lambda^c\right) T^c. \tag{3.8}$$

Here, it is important to mention that each term in the Lagrangian can be expressed as a polynomial of the 1 body operators, i.e., each operator has well defined $1/N_c$ expansion. The flavor indices in the Lagrangian run from 1 to 9 because the nonet of pion is considered.

$\mathcal{M}_{\text{hyperfine}}$ represents the spin splittings of the tower of baryon with spin $1/2, ..., N_c/2$ in the flavor representations. Moreover, baryon vector and axial vector currents are given by

$$\mathcal{V}^0 = \frac{1}{2}\left(\xi\partial^0\xi^\dagger + \xi^\dagger\partial^0\xi\right), \quad \mathcal{A}^k = \frac{i}{2}\left(\xi\nabla^k\xi - \xi^\dagger\nabla^k\xi\right). \tag{3.9}$$

In those expressions, $\xi = \exp\left[i\Pi(x)/f\right]$, where $\Pi(x)$ denotes the nonet of Goldstone bosons fields as

$$\Pi(x) = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}(\eta + \eta') & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}(\eta+\eta') & K^0 \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{6}}(2\eta + \eta'), \end{pmatrix} \tag{3.10}$$

and $f \approx 93$MeV is the meson decay constant.

The baryon mass operator $\mathcal{M}$ has its $1/N_c$ expansion as [15, 23]

$$\mathcal{M} = m_0 N_c \mathbb{1} + \sum_{N_c - 1}^{n=2,4} m_n \frac{1}{N_c^{n-1}} J^n, \tag{3.11}$$

where m_n are unknown coefficients. Since the first term is removed by heavy baryon chiral perturbation theory [21] and considering the physical value $N_c = 3$, the mass operator is defined as the hyperfine mass given by

$$\mathcal{M}_{\text{hyperfine}} = \frac{m_2}{N_c} J^2. \tag{3.12}$$

Finally, there is one more operator on the Lagrangian in (3.7), A^{kc} is the baryon axial vector current. It is a spin-1 object, an octet under $SU(3)$, odd under time reversal, and its $1/N_c$ expansion is [5]

$$\mathcal{A}^{kc} = a_1 G^{kc} + \sum_{n=2,3}^{N_c} b_n \frac{1}{N_c^{n-1}} \mathcal{D}_n^{kc} + \sum_{n=3,5}^{N_c} c_n \frac{1}{N_c^{n-1}} \mathcal{O}_n^{kc}, \tag{3.13}$$

where $\mathcal{O}_n^{kc}$ are completely off-diagonal operators with nonzero matrix elements only between states with a different spin, and $\mathcal{D}_n^{kc}$ are diagonal operators with nonzero matrix elements only between states with the same spin. The first terms in the expansion follow

$$\begin{aligned}
\mathcal{D}_2^{kc} &= J^K T_c, \\
\mathcal{O}_2^{kc} &= \epsilon^{ijk} \{J^i, G^{jc}\}, \\
\mathcal{D}_3^{kc} &= \{J^k, \{J^r, G^{rc}\}\}, \\
\mathcal{O}_3^{kc} &= \{J^2, G^{kc}\} - \frac{1}{2}\{J^k, \{J^r, G^{rc}\}\}.
\end{aligned}$$

For higher order terms, there are two recursive formulas $\mathcal{D}_n^{kc} = \{J^2, D_2^{kc}\}$ and $\mathcal{O}_n^{kc} = \{J^2, \mathcal{O}_{n-2}^{kc}\}$ with $n \geq 4$. From the definitions for the operator $\mathcal{O}_n$, it is forbidden in the expansion for axial vector current since that are even under time reversal. Additionally, the coefficients a_1, b_n, and c_n have expansion in powers of $1/N_c$ and are order unity.

Considering the physical value $N_c = 3$ the series for A^{kc} can be truncated as

$$A^{kc} = a_1 G^{kc} + b_2 \frac{1}{N_c} \mathcal{D}_2^{kc} + b_3 \frac{1}{N_c} \mathcal{D}_3^{kc} + c_3 \frac{1}{N_c} \mathcal{O}_3^{kc}. \tag{3.14}$$

and this expansion extends up to 3-body operators. For octet baryons, the axial vector couplings are given by $g_A \approx 1.27$ and $g_V = 1$, as defined in β-decay experiments for neutron decay. Analogously, the baryon axial current A^k is a spin-1 object and a singlet under

$SU(3)$, so it doesn't contain flavor indices, and its $1/N_c$ expansion is [20]:

$$A^k = \sum_{n=1,3}^{N_c} b_n^{1,1} \frac{1}{N_c^{n-1}} \mathcal{D}_n^k, \tag{3.15}$$

where $\mathcal{D}_1^k = J^k$ and follow the recursive formula $\mathcal{D}_1^k = \{J^2, \mathcal{D}_{2m-1}^k\}$ for $m \geq 1$. In these expressions, the superscript on the coefficients for A^k indicates that they are related to the baryon singlet current. Again, considering the physical limit $N_c = 3$, the expansion can be truncated to get

$$A^k = b_1^{1,1} J^k + b_3^{1,1} \frac{1}{N_c^2} \{J^2, J^k\}. \tag{3.16}$$

The complete analysis of the renormalization process for the baryon axial vector current has been performed in [24].

3.4 Projection Operators for $SU(2) \otimes SU(3)$ Spin-Flavor Algebra

Through the $1/N_c$ expansion, it is possible to expand any QCD operator as a combination of 1-body operator. Supposing that the most general 1-body operators are written as X^{ia}, whose indices transform as a spin-1 object and under the adjoint representation of the $SU(N_f)$. Then, the most general 2-body operator must contain two spin and two flavor indices, the same applies for a 3 body operator, and so on.

In different cases, there are tensor operators that include three spin and flavor free indices given by the processes involved in their respective Feynman diagrams. For example, the renormalization of axial baryon current or the baryon-meson scattering process.

Since it is not always possible to compute the matrix elements for a certain combination of operators, the usual way to deal with them is by expanding those operators in terms of a basis composed of operators with the same or less number of bodies. However, this decomposition is not trivial, because there could be a large number of operators in the basis. So, a systematic method to classify the operators according to certain criteria is needed.

Fortunately, the projector technique developed in Chapter 2 is helpful for these cases. Projection operators can decompose any n-body spin-flavor operator in components related to a particular irreducible representation. Being that some hadron properties or processes are described by particular representations, the projector technique simplifies the computations and selection for operators that appear in the different decompositions for each case.

For a general tensor operator with two spin and two flavor indices $X_1^{i_1 a_1} X_2^{i_2 a_2}$, it is possible to decompose them using the projectors from eqs. (2.28) to (2.32). As the spin components are independent of the flavor components, there are two different sets of projection operators. The first one, assumes $N = 2$ for spin, so the projectors take the form given in eqs. (2.40), (2.43) and (2.44). The second case considers $N = N_f$ and it works for any value of $N_f \geq 2$. However, for tensor operators with three spin or three flavor indices, the computations are more complicated. Since the flavor indices in those operators transform under the representation $adj \otimes adj \otimes adj$, and this product has more representations in its decomposition, the projector will contain higher powers of Casimir and the computation for each one is not convenient for the application purpose. An example of the projectors given by a component of this product appears in [25].

To implement the projection technique to tensor operators with three or more flavor indices in a suitable way, i.e. avoiding the inconvenience of large computations due to higher powers of Casimir, it is necessary to take particular values for N_f.

In the mixed formalism of chiral perturbation theory and the $1/N_c$ expansion, the theory is restricted to the light quark sector, so $N_f = 3$. Therefore, the spin flavor group is $SU(2) \times SU(3)$, and the adjoint representation for $SU(3)$ flavor group is **8**.

Now, considering $N_f = 3$, an object with two flavor indices transforms under a combination of the representations given by the tensor product

$$\mathbf{8} \otimes \mathbf{8} = \mathbf{1} \oplus 2(\mathbf{8}) \oplus (\mathbf{10} \oplus \bar{\mathbf{10}}) \oplus \mathbf{27}. \tag{3.17}$$

In this way, the projectors are the same as eqs. (2.28) to (2.32), considering $N = 3$. With the same value for N_f, an object with three flavor indices requires the decomposition for the tensor product:

$$\mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8} = 2(\mathbf{1}) \oplus 8(\mathbf{8}) \oplus 4(\mathbf{10} + \bar{\mathbf{10}}) 6(\mathbf{27}) \oplus 2(\mathbf{35} + \bar{\mathbf{35}}) \oplus \mathbf{64}. \tag{3.18}$$

For this case, the eigenvalues for the Casimir are:

$$
\begin{aligned}
c^{\mathbf{1}}_{flavor} &= 0, \quad & c^{\mathbf{8}}_{flavor} &= 3, \quad & c^{\mathbf{27}}_{flavor} &= 8, \\
c^{\mathbf{64}}_{flavor} &= 15, \quad & c^{\mathbf{10}+\bar{\mathbf{10}}}_{flavor} &= 6, \quad & c^{\mathbf{35}+\bar{\mathbf{35}}}_{flavor} &= 12,
\end{aligned}
$$

then, the construction of the projectors reads the same formalism as in the case of four indices, as follows:

$$
\begin{aligned}
\left[P^{(m)}_{flavor}\right]^{c_1c_2c_3b_1b_2b_3} &= \prod_{i=1}^{5}\left[\left(\frac{\hat{C}_{flavor} - c^{n_i}_{flavor}\mathbb{1}}{c^{m}_{flavor} - c^{n_i}_{flavor}}\right)\right]^{c_1c_2c_3b_1b_2b_3} \\
=& \frac{1}{\prod_{i=1}^{5}(c^{m}_{flavor} - c^{n_i}_{flavor})}\Big[\beta_1\delta^{c_1b_1}\delta^{c_2b_2}\delta^{c_3b_3}\delta^{c_4b_4}\delta^{c_5b_5} + \beta_2[C_{flavor}]^{c_1c_2c_3b_1b_2b_3} \\
&+ \beta_3\left[C^2_{flavor}\right]^{c_1c_2c_3b_1b_2b_3} + \beta_4\left[C^3_{flavor}\right]^{c_1c_2c_3b_1b_2b_3} + \beta_5\left[C^4_{flavor}\right]^{c_1c_2c_3b_1b_2b_3} \\
&+ \left[C^5_{flavor}\right]^{c_1c_2c_3b_1b_2b_3}\Big],
\end{aligned}
\tag{3.19}
$$

here, the coefficients are

$$
\beta_1 = -c^{n_1}_{flavor}c^{n_2}_{flavor}c^{n_3}_{flavor}c^{n_4}_{flavor}c^{n_5}_{flavor},
$$

$$
\begin{aligned}
\beta_2 =& c^{n_1}_{flavor}c^{n_2}_{flavor}c^{n_3}_{flavor}c^{n_4}_{flavor} + c^{n_1}_{flavor}c^{n_2}_{flavor}c^{n_3}_{flavor}c^{n_5}_{flavor} \\
&+ c^{n_1}_{flavor}c^{n_2}_{flavor}c^{n_4}_{flavor}c^{n_5}_{flavor} + c^{n_1}_{flavor}c^{n_3}_{flavor}c^{n_4}_{flavor}c^{n_5}_{flavor} \\
&+ c^{n_2}_{flavor}c^{n_3}_{flavor}c^{n_4}_{flavor}c^{n_5}_{flavor},
\end{aligned}
$$

$$
\begin{aligned}
\beta_3 =& -(c^{n_1}_{flavor}c^{n_2}_{flavor}c^{n_3}_{flavor} + c^{n_1}_{flavor}c^{n_2}_{flavor}c^{n_4}_{flavor} + c^{n_1}_{flavor}c^{n_2}_{flavor}c^{n_5}_{flavor} \\
&+ c^{n_1}_{flavor}c^{n_3}_{flavor}c^{n_4}_{flavor} + c^{n_1}_{flavor}c^{n_3}_{flavor}c^{n_5}_{flavor} + c^{n_1}_{flavor}c^{n_4}_{flavor}c^{n_5}_{flavor} \\
&+ c^{n_2}_{flavor}c^{n_3}_{flavor}c^{n_4}_{flavor} + c^{n_2}_{flavor}c^{n_3}_{flavor}c^{n_5}_{flavor} + c^{n_2}_{flavor}c^{n_4}_{flavor}c^{n_5}_{flavor} \\
&+ c^{n_3}_{flavor}c^{n_4}_{flavor}c^{n_5}_{flavor}),
\end{aligned}
$$

$$
\begin{aligned}
\beta_4 =& c^{n_1}_{flavor}c^{n_2}_{flavor} + c^{n_1}_{flavor}c^{n_3}_{flavor} + c^{n_1}_{flavor}c^{n_4}_{flavor} + c^{n_1}_{flavor}c^{n_5}_{flavor} + c^{n_2}_{flavor}c^{n_3}_{flavor} \\
&+ c^{n_2}_{flavor}c^{n_4}_{flavor} + c^{n_2}_{flavor}c^{n_5}_{flavor} + c^{n_3}_{flavor}c^{n_4}_{flavor} + c^{n_3}_{flavor}c^{n_5}_{flavor} + c^{n_4}_{flavor}c^{n_4}_{flavor},
\end{aligned}
$$

$$
\beta_5 = -(c^{n_1}_{flavor} + c^{n_2}_{flavor} + c^{n_3}_{flavor} + c^{n_4}_{flavor} + c^{n_5}_{flavor}),
$$

and the Casimir is defined by

$$[C_{flavor}]^{c_1c_2c_3b_1b_2b_3} = [T^a_A]^{c_1c_2c_3d_1d_2d_3}\,[T^a_A]^{d_1d_2d_3b_1b_2b_3}\,, \tag{3.20}$$

where the six indices generators are

$$\begin{aligned}[T^a_A]^{c_1c_2c_3b_1b_2b_3} &= [T^a_A]^{c_1c_2b_1b_2}\,\delta^{c_3b_3} + \delta^{c_1b_1}\delta^{c_2b_2}\,[T^a_A]^{c_3b_3} \\ &= -i\left(f^{ac_1b_1}\delta^{c_2b_2} + \delta^{c_1b_1}f^{ac_2b_2}\right)\delta^{c_3b_3} + \delta^{c_1b_1}\delta^{c_2b_2}\left(-if^{ac_3b_3}\right) \\ &= -i\left(\delta^{c_1b_1}\delta^{c_2b_2}f^{ac_3b_3} + \delta^{c_2b_2}\delta^{c_3b_3}f^{ac_1b_1} + \delta^{c_1b_1}\delta^{c_3b_3}f^{ac_2b_2}\right).\end{aligned} \tag{3.21}$$

Thus, explicit Casimir reads

$$\begin{aligned}[C_{flavor}]^{c_1c_2c_3b_1b_2b_3} = &-\left(\delta^{c_1d_1}\delta^{c_2d_2}f^{ac_3d_3} + \delta^{c_2d_2}\delta^{c_3d_3}f^{ac_1d_1} + \delta^{c_1d_1}\delta^{c_3d_3}f^{ac_2d_2}\right) \\ &\times\left(\delta^{d_1b_1}\delta^{d_2b_2}f^{ad_3b_3} + \delta^{d_2b_2}\delta^{d_3b_3}f^{ad_1b_1} + \delta^{d_1b_1}\delta^{d_3b_3}f^{ad_2b_2}\right),\end{aligned} \tag{3.22}$$

and after some reductions, it takes the form:

$$\begin{aligned}[C_{flavor}]^{c_1c_2c_3b_1b_2b_3} = [&6\delta^{c_1b_1}\delta^{c_2b_2}\delta^{c_3b_3} - 2\delta^{c_1b_1}f^{ac_2b_2}f^{ac_3b_3} \\ &- 2\delta^{c_2b_2}f^{ac_1b_1}f^{ac_3b_3} - 2\delta^{c_3b_3}f^{ac_1b_1}f^{ac_2b_2}].\end{aligned} \tag{3.23}$$

Even with the fixed value for $N_f = 3$, it is still difficult to use symbolic algebra to compute operations between projectors with three indices. To cope with these difficulties, it is better to implement a numerical procedure in this case. So, for this case, a matrix numerical method has been created.

Each projector or Casimir operator with 6 flavor indices contains 8^6 elements because each index can take 8 different values. To construct the projectors, Casimir operators have all or half of their indices contracted, and the projectors are applied over spin flavor operators with three flavor indices. Therefore, half of projector indices will be always contracted as

$$\left[P^{(m)}_{flavor}\right]^{c_1c_2c_3b_1b_2b_3}\mathcal{O}^{b_1b_2b_3}_{3-body} \quad \text{or} \quad C^{c_1c_2c_3b_1b_2b_3}_{flavor}C^{b_1b_2b_3a_1a_2a_3}_{flavor}.$$

To simplify the computations, it is possible to collect the first three indices (c_1, c_2, c_3) and the last three indices (b_1, b_2, b_3) of Casimir and projectors in only two indices C and B respectively, and these new indices run over $8^3 = 512$ possible values. This modification transforms the tensor projection operators into a matrix representation, and projectors are now described as 512×512 matrices. Similarly, 3-body operators with three flavor indices can be represented as vectors with 512 entries. Thus, instead of performing the index contractions, it is only needed to compute matrix multiplications.

It is important to mention that this modification does not represent a loss of information but simplifies the computation of numerical application of projectors over operators.

Chapter 4

Baryon-Meson Scattering

In particle physics, the baryon-meson scattering process is an interesting topic that has been treated using different formalisms. Since the first efforts from Deloff and Lipkin [26, 27], who worked on this scattering process in a quark model before the development of more sophisticated methods for baryons and mesons, the problem has obtained important advancements in the recent decades in the phenomenological and experimental bent [28].

In the context of the effective field theories, baryon chiral perturbation theory (BChPT) has obtained different phenomenological results as presented in Ref. [29], as well as HBChPT which performed recent computations to orders $\mathcal{O}(p^3)$ and $\mathcal{O}(p^4)$ [30, 31]. Besides perturbative methods, different challenges must be treated by first-principles QCD calculations, so the best approach for them is lattice QCD, which also has reported advances in the computation of scattering amplitudes for baryon-meson systems [32].

One of the most outstanding approaches to tackling the baryon-meson scattering problem is large N_c QCD. The first work using this formalism to describe the process was made by t'Hooft and Witten [3, 4], where the baryon-meson couplings were set out for the first time. From the analysis of the large N_c counting rules for baryon-meson scattering, Witten deduced that the amplitude for this process is of order one at fixed energy.

Subsequently, Dashen, Jenkins, and Manohar obtained important results on baryon static properties considering the large N_c power counting rules for multimeson-baryon0baryon scattering amplitudes [16]. Later on, Flores-Mendieta, Hofmann, and Jenkins studied tree-level amplitudes for baryon-meson scattering and deduced generalized large N_c consistency conditions, that are valid to all orders in the baryon mass splitting [33]. This mass splitting is given by $\Delta \equiv M_T - M_B$, where M_T and M_B are the baryon decuplet and baryon octet masses, respectively.

In this chapter, the 1/Nc expansion of the baryon operator whose matrix elements between baryon states yield the scattering amplitude is constructed. Also, the most complete form of this amplitude is obtained by accounting for the decuplet-octet baryon mass difference explicitly.

4.1 Baryon-meson scattering amplitude at tree level

The analytical computation of tree-level amplitude of baryon-meson scattering describes the process

$$B(p) + \pi^a(k) \longrightarrow B'(p') + \pi^b(k'). \tag{4.1}$$

The Feynman diagrams that represent the scattering appear in Fig. 4.1.

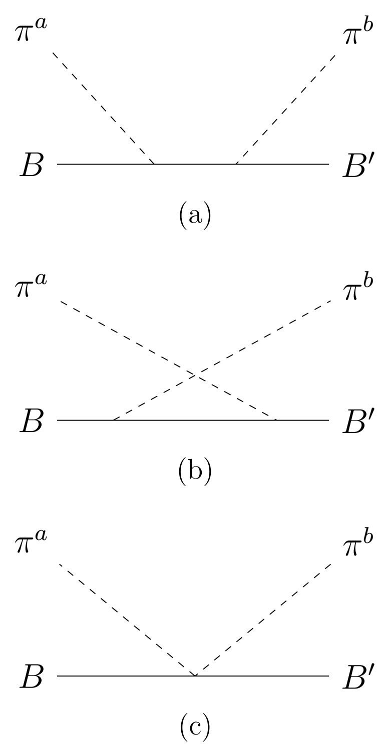


Figure 4.1: Feynman diagrams for the tree-level scattering process (4.1)

In (4.1), B and B' denote the incoming and outgoing baryons, respectively, with momenta p and p'. π represents the nine pseudo scalar mesons π, K, η and η' (nonet of mesons) of momenta $k = (k^0, k^1, k^2, k^3)$ and $k' = (k'^0, k'^1, k'^2, k'^3)$, respectively, and indices a and b indicates the flavor for the incoming and outgoing mesons. Additionally, soft mesons with energies of order unity are included in the computations.

Since the analysis considers the decuplet and octet of baryons, the goal of this work is to evaluate the scattering amplitude at tree level, considering the effects of the baryon mass splitting Δ.

4.1.1 Scattering amplitude from Fig. 4.1 (a) and (b)

The baryon operator that describes the tree level amplitude for the baryon-meson scattering process, represented in the diagram (a) and (b) from Fig. 4.1, in the rest frame of the initial baryon B, is given by [33]

$$A_{\text{tree}}^{ab} = -\frac{1}{f^2} k^i k'^j \left[\frac{1}{k^0} \sum_{n=0}^{\infty} \frac{1}{k^{0n}} [A^{jb}, \underbrace{[\mathcal{M}, [\mathcal{M}, \ldots [\mathcal{M}}_{n \text{ insertions}}, A^{ia} \underbrace{]\ldots]]]}\right], \tag{4.2}$$

where f is the pion decay constant, A^{ia} is the baryon axial vector current and $\mathcal{M}$ is the baryon mass operator. All of them are defined in Chapter 3, and assuming the physical value $N_c = 3$, so their $1/N_c$ expansions are truncated. Then, as $\mathcal{M}$ is now $\mathcal{M}_{\text{hyperfine}}$ from (3.11), m_2 can be set as $m_2 = \Delta$. The numerical average value for this constant is $\Delta = 0.231$GeV.

The first terms expanded in the series given by (4.2) look like

$$\begin{aligned} A_{\text{tree}}^{ab} = -\frac{1}{f^2} k^i k'^j \Bigg[& \frac{1}{k^0}[A^{jb}, A^{ia}] + \frac{1}{k^{0^2}}[A^{jb}, [\mathcal{M}, A^{ia}]] \\ & + \frac{1}{k^{0^3}}[A^{jb}, [\mathcal{M}, [\mathcal{M}, A^{ia}]]] + \ldots \Bigg]. \end{aligned} \tag{4.3}$$

The amplitude A_{tree}^{ab} is constrained to be at most $\mathcal{O}(1)$ in the large N_c limit, and consequently there are the next consistency conditions [15, 33]

$$[A^{jb}, A^{ia}] \leq \mathcal{O}(N_c), \tag{4.4a}$$

$$[A^{jb}, [\mathcal{M}, A^{ia}]] \leq \mathcal{O}(N_c), \tag{4.4b}$$

$$[A^{jb}, [\mathcal{M}, [\mathcal{M}, A^{ia}]]] \leq \mathcal{O}(N_c), \tag{4.4c}$$

$$\vdots$$

where k^0, f, and Δ are orders $\mathcal{O}(1)$, $\mathcal{O}(\sqrt{N_c})$, and $\mathcal{O}(N_c^{-1})$ in that limit, respectively.

Regarding symmetry aspects of A^{ab}_{tree}, it is a spin-zero object and contains two adjoint (octet) indices related to flavor. The tensor product of two adjoint representations can be split into a symmetric $(\mathbf{8}\otimes\mathbf{8})_S$ and an antisymmetric $(\mathbf{8}\otimes\mathbf{8})_S$ product, which can be decomposed into irreps as

$$(\mathbf{8}\otimes\mathbf{8})_S = \mathbf{1}\oplus\mathbf{8}\oplus\mathbf{27}, \tag{4.5a}$$

$$(\mathbf{8}\otimes\mathbf{8})_A = \mathbf{8}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}. \tag{4.5b}$$

In order to take advantage of the symmetry transformation properties of $A^{ab_{\text{tree}}}$ under the spin flavor symmetry group $SU(2)\times SU(3)$, the projection operators from chapter 2 are introduced in this problem.

For a general flavor value, the projectors from eqs. (2.28) to (2.32) can be adapted to the irreps from (4.5), to obtain

$$\left[\mathcal{P}^{(1)}\right]^{abcd} = \frac{1}{N_f^2-1}\delta^{ab}\delta^{cd}, \tag{4.6}$$

$$\left[\mathcal{P}^{(8)}\right]^{abcd} = \frac{N_f}{N_f^2-4}d^{abe}d^{cde}, \tag{4.7}$$

$$\left[\mathcal{P}^{(27)}\right]^{abcd} = \frac{1}{2}(\delta^{ac}\delta^{bd}+\delta^{bc}\delta^{ad}) - \frac{1}{N_f^2-1}\delta^{ab}\delta^{cd} - \frac{N_f}{N_f^2-4}d^{abe}d^{cde}, \tag{4.8}$$

$$\left[\mathcal{P}^{(8_A)}\right]^{abcd} = \frac{1}{N_f}f^{abe}f^{cde}, \tag{4.9}$$

and

$$\left[\mathcal{P}^{(10+\bar{1}0)}\right]^{abcd} = \frac{1}{2}(\delta^{ac}\delta^{bd}-\delta^{bc}\delta^{ad}) - \frac{1}{N_f}f^{abe}f^{cde}, \tag{4.10}$$

that satisfy the completeness relation

$$\left[\mathcal{P}^{(1)}+\mathcal{P}^{(8)}+\mathcal{P}^{(27)}+\mathcal{P}^{(8_A)}+\mathcal{P}^{(10+\bar{1}0)}\right]^{abcd} = \delta^{ac}\delta^{bd}. \tag{4.11}$$

Defining $\left[\mathcal{P}^{(m)}A_{\text{tree}}\right]^{ab}$ as the component projected out from A^{ab}_{tree}. It transforms under the particular flavor representation of dimension m from the representations listed on (4.5). Nevertheless, for computational purposes, it is more helpful to collect the operators $\left[\mathcal{P}^{(m)}A_{\text{tree}}\right]^{ab}$ using the transformation properties under the interchange of a and b. In this way, there is a symmetric projector $\left[\mathcal{P}^{(1)}+\mathcal{P}^{(8)}+\mathcal{P}^{(27)}\right]^{ab}$ and antisymmetric projector $\left[\mathcal{P}^{(8_A)}+\mathcal{P}^{(10+\bar{1}\bar{0})}\right]^{ab}$ that pull out the symmetric and antisymmetric parts of A^{ab}_{tree}.

4.1.2 Explicit form of the scattering amplitude at tree level

The analysis for the baryon-meson scattering amplitude in the $1/N_c$ expansion can be constructed in a similar way to other baryon properties or couplings, by expanding in terms of spin-flavor generators. In this case, a detailed calculation can be performed by considering the first summands of (4.3).

By simple inspection, the first term is proportional to $\left[A^{jb},A^{ia}\right]$, and considering the expansion for $N_c=3$, this term retains up to 5-body operators. Since the next summands contain a mass term $\mathcal{M}$ and an extra commutator for each additional term considered. The next two terms add up 6 and 7-body operators, respectively. In conclusion, A^{ab}_{tree} contains at most 7-body operators, if the three first summands are considered, and a complete 7-body operator basis is needed to construct the $1/N_c$ expansion of the baryon-meson scattering amplitude.

The matrix elements of the operator A^{ab}_{tree} in the transition from the $SU(6)$ baryon states for the incoming pair B and π^a to the outgoing pair B' and π^b, describe the scattering amplitude at tree level as

$$\mathcal{A}_{\text{tree}}(B+\pi^a\to B'+\pi^b)\equiv\langle B'\pi^b|A^{ab}_{\text{tree}}|B\pi^a\rangle. \tag{4.12}$$

The flavors related to mesons are given by $\left\{\frac{1-i2}{\sqrt{2}},3,\frac{1+i2}{\sqrt{2}},\frac{4-i5}{\sqrt{2}},\frac{6-i7}{\sqrt{2}},\frac{4+i5}{\sqrt{2}},\frac{6+i7}{\sqrt{2}},8\right\}$ for $\{\pi^+,\pi^0,\pi^-,K^+,K^0,K^-,\overline{K}^0,\eta\}$, respectively. As an example, $\mathcal{A}_{\text{tree}}(p+\pi^-\to n+\pi^0)$ is given by $\langle n\pi^0|A^{13}_{\text{tree}}+iA^{23}_{\text{tree}}|p\pi^-\rangle/\sqrt{2}$. For simplicity, the η' is not considered in these expressions, but the analysis is straightforwards by using the baryon axial current $A^i\equiv A^{i9}$, which can be written in terms of the operators $G^{i9}=\frac{1}{\sqrt{6}}J^i$ and $T^9=\frac{N_c}{\sqrt{6}}\mathbb{1}$ [20].

Thus, the scatting amplitude process for the diagrams Fig.4.1(a,b) has an expansion as

$$\mathcal{A}_{\text{tree}}(B+\pi^a \to B'+\pi^b) = -\frac{1}{f^2 k^0}\sum_{m=1}^{139}(c_m^{(\text{s})}+c_m^{(\text{a})})k^i k'^j \langle B'\pi^b|S_m^{(ij)(ab)}|B\pi^a\rangle, \tag{4.13}$$

where $S_m^{(ij)(ab)}$ $(m = 1,\dots,139)$ constitute a basis of spin-2 baryon operators, which are linearly independent, with two adjoint indices and $c_m^{(\text{s})}$ and $c_m^{(\text{a})}$ are well-defined coefficients which come along with the symmetric and antisymmetric components of A^{ab}. The operators $S_m^{(ij)(ab)}$, and coefficients $c_m^{(\text{s})}$ and $c_m^{(\text{a})}$ are listed in Appendix B.

The equation (4.13) for the baryon-meson amplitude, as mentioned above, just describes the first two diagrams of Fig.4.1. Diagram (c), illustrates the 2-meson-baryon-baryon contact interactions, which contributes to the amplitude with the term

$$A^{ab}_{\text{vertex}} = -\frac{1}{2f^2}(2k^0 + M - M')f^{abc}T^c, \tag{4.14}$$

where M is the mass of the initial baryon and M' the mass of the final baryon [33]. Since this contribution is antisymmetric under the interchange of a and b, and after applying the projection operators, $\left[\mathcal{P}^{(8_A)}A_{\text{vertex}}\right]^{ab}$ is the only component that does not vanish, which implies that this term just contributes to the octet component of the total scattering amplitude. Also, A^{ab}_{vertex} is order $\mathcal{O}(1)$, so this operator together with A^{ab}_{tree} yield the leading order $\mathcal{O}(1)$ scattering amplitude for baryons with spin J in the limit of exact $SU(3)$ flavor symmetry.

4.2 Example: $N\pi \to N\pi$ scattering amplitude

The presented formalism can be applied to processes of the form $B+\pi^a \to B'+\pi^b$ according to the Gell-Mann-Nishijima scheme, which implies that the total strangeness must be equal on both sides of the process, and the baryons in the process can be either octet or decuplet. As an example, the nucleon-pion scattering process will be analyzed.

The particles involved in the process have different values for weak isospin T, for pions $T = 1$, and for baryons $T = \frac{1}{2}$. Then, following the usual rules for the addition of angular momenta, both particles can be combined in single states with $T = \frac{1}{2}$ or $T = \frac{3}{2}$. The possible states are listed in Table 4.1.

	$T=\frac{3}{2}$	$T=\frac{1}{2}$
$T_3=+\frac{1}{2}$	$\sqrt{\frac{1}{3}}\lvert n\pi^+\rangle + \sqrt{\frac{2}{3}}\lvert p\pi^0\rangle$	$\sqrt{\frac{2}{3}}\lvert n\pi^+\rangle - \sqrt{\frac{1}{3}}\lvert p\pi^0\rangle$
$T_3=-\frac{1}{2}$	$\sqrt{\frac{2}{3}}\lvert n\pi^0\rangle + \sqrt{\frac{1}{3}}\lvert p\pi^-\rangle$	$\sqrt{\frac{1}{3}}\lvert n\pi^0\rangle - \sqrt{\frac{2}{3}}\lvert p\pi^-\rangle$
$T_3=-\frac{3}{2}$	$\lvert n\pi^-\rangle$	

Table 4.1: Allowed states for pion-nucleon system

Applying the usual Clebsh-Gordan technique, the scattering amplitude for (4.1) can be decomposed into two different amplitudes $\mathcal{A}^{(T)}$, for $T=\frac{1}{2},\frac{3}{2}$. To compute the amplitudes, the s-channel isospin eigenstates for the states listed on Table 4.1 are

$$|p\pi^+\rangle = \left|\frac{3}{2},+\frac{3}{2}\right\rangle, \tag{4.15}$$

$$|n\pi^+\rangle = \sqrt{\frac{1}{3}}\left|\frac{3}{2},+\frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|\frac{1}{2},+\frac{1}{2}\right\rangle, \tag{4.16}$$

$$|p\pi^0\rangle = \sqrt{\frac{2}{3}}\left|\frac{3}{2},+\frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\left|\frac{1}{2},+\frac{1}{2}\right\rangle, \tag{4.17}$$

$$|n\pi^0\rangle = \sqrt{\frac{2}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle + \sqrt{\frac{1}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle, \tag{4.18}$$

$$|p\pi^-\rangle = \sqrt{\frac{1}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle, \tag{4.19}$$

$$|n\pi^-\rangle = \left|\frac{3}{2},-\frac{3}{2}\right\rangle, \tag{4.20}$$

then, the amplitudes are [34]

$$\begin{aligned}
\mathcal{A}_{\text{tree}}(p+\pi^+ \to p+\pi^+) &= \mathcal{A}_{\text{tree}}(n+\pi^- \to n+\pi^-) = \mathcal{A}^{(3/2)},\\
\mathcal{A}_{\text{tree}}(p+\pi^- \to p+\pi^-) &= \mathcal{A}_{\text{tree}}(n+\pi^+ \to n+\pi^+) = \frac{1}{3}\mathcal{A}^{(3/2)} + \frac{2}{3}\mathcal{A}^{(1/2)},\\
\mathcal{A}_{\text{tree}}(p+\pi^0 \to p+\pi^0) &= \mathcal{A}_{\text{tree}}(n+\pi^0 \to n+\pi^0) = \frac{2}{3}\mathcal{A}^{(3/2)} + \frac{1}{3}\mathcal{A}^{(1/2)},\\
\sqrt{2}\mathcal{A}_{\text{tree}}(p+\pi^- \to n+\pi^0) &= \sqrt{2}\mathcal{A}_{\text{tree}}(n+\pi^+ \to p+\pi^0) = \frac{2}{3}\mathcal{A}^{(3/2)} - \frac{2}{3}\mathcal{A}^{(1/2)}
\end{aligned} \tag{4.21}$$

Moreover, there is a different set of invariant amplitudes, which can be defined in terms of $\mathcal{A}^{(3/2)}$ and $\mathcal{A}^{(1/2)}$ for the $N\pi$ system as [35]

$$\mathcal{A}^{(+)} = \frac{2}{3}\mathcal{A}^{(3/2)} + \frac{1}{3}\mathcal{A}^{(1/2)}, \qquad \mathcal{A}^{(-)} = -\frac{1}{3}\mathcal{A}^{(3/2)} + \frac{1}{3}\mathcal{A}^{(1/2)}. \tag{4.22}$$

All the non-trivial matrix elements $k^i k'^j \langle B'\pi^b | S_m^{(ij)(ab)} | B\pi^a \rangle$ for nucleon-pion processes appear on Tables tables B.1 and B.2. As an interesting result, the symmetric component of the amplitude is proportional to $\mathbf{k}\cdot\mathbf{k}'$, similarly the antisymmetry component is proportional to the third component $i(\mathbf{k}\times\mathbf{k}')$, denoted as $i(\mathbf{k}\times\mathbf{k}')^3$.

4.2.1 Tree-level scattering amplitude for Fig. 4.1 (a,b)

Combining the obtained results for matrix elements and the coefficients related to each one, the scattering amplitudes for $N\pi$ processes at tree-level can be written as

$$\begin{aligned}
&f^2 k^0 \mathcal{A}_{\text{tree}}(p+\pi^+ \to p+\pi^+) \\
&= \Bigg[-\frac{25}{72}a_1^2 - \frac{5}{36}a_1 b_2 - \frac{25}{108}a_1 b_3 - \frac{1}{72}b_2^2 - \frac{5}{108}b_2 b_3 - \frac{25}{648}b_3^2 \qquad (4.23)\\
&\qquad + \frac{2}{9}\left[1 - \frac{2\Delta}{k^0} + \frac{\Delta^2}{k^{02}}\right]\left[a_1^2 + a_1 c_3 + \frac{1}{4}c_3^2\right]\Bigg]\mathbf{k}\cdot\mathbf{k}' \\
&\quad + \Bigg[\frac{25}{72}a_1^2 + \frac{5}{36}a_1 b_2 + \frac{25}{108}a_1 b_3 + \frac{1}{72}b_2^2 + \frac{5}{108}b_2 b_3 + \frac{25}{648}b_3^2 \\
&\qquad - \frac{2}{9}\left[1 - \frac{1}{2}\frac{\Delta}{k^0} + \frac{\Delta^2}{k^{02}}\right]\left[a_1^2 + a_1 c_3 + \frac{1}{4}c_3^2\right]\Bigg] i(\mathbf{k}\times\mathbf{k}')^3 + \mathcal{O}\left[\frac{\Delta^3}{k^{03}}\right] \\
&= f^2 k^0 \mathcal{A}_{\text{tree}}(n+\pi^- \to n+\pi^-), \qquad (4.24)
\end{aligned}$$

$$\begin{aligned}
&f^2 k^0 \mathcal{A}_{\text{tree}}(p+\pi^- \to p+\pi^-) \\
&= \Bigg[\frac{25}{72}a_1^2 + \frac{5}{36}a_1 b_2 + \frac{25}{108}a_1 b_3 + \frac{1}{72}b_2^2 + \frac{5}{108}b_2 b_3 + \frac{25}{648}b_3^2 \qquad (4.25)\\
&\qquad - \frac{2}{9}\left[1 + \frac{2\Delta}{k^0} + \frac{\Delta^2}{k^{02}}\right]\left[a_1^2 + a_1 c_3 + \frac{1}{4}c_3^2\right]\Bigg]\mathbf{k}\cdot\mathbf{k}'
\end{aligned}$$

$$
\begin{aligned}
&+ \left[\frac{25}{72}a_1^2 + \frac{5}{36}a_1b_2 + \frac{25}{108}a_1b_3 + \frac{1}{72}b_2^2 + \frac{5}{108}b_2b_3 + \frac{25}{648}b_3^2 \right. \\
&\quad \left. -\frac{2}{9}\left[1 + \frac{1}{2}\frac{\Delta}{k^0} + \frac{\Delta^2}{{k^0}^2}\right]\left[a_1^2 + a_1c_3 + \frac{1}{4}c_3^2\right]\right] i(\mathbf{k}\times\mathbf{k}')^3 + \mathcal{O}\left[\frac{\Delta^3}{{k^0}^3}\right] \\
&= f^2k^0\mathcal{A}_{\text{tree}}(n + \pi^+ \to n + \pi^+),
\end{aligned} \tag{4.26}
$$

$$
\begin{aligned}
&f^2k^0\mathcal{A}_{\text{tree}}(p + \pi^0 \to p + \pi^0) \\
&= -\frac{4}{9}\frac{\Delta}{k^0}\left[a_1^2 + a_1c_3 + \frac{1}{4}c_3^2\right]\mathbf{k}\cdot\mathbf{k}' + \left[\frac{25}{72}a_1^2 + \frac{5}{36}a_1b_2\right. \qquad (4.27) \\
&\quad + \frac{25}{108}a_1b_3 + \frac{1}{72}b_2^2 + \frac{5}{108}b_2b_3 + \frac{25}{648}b_3^2 \\
&\quad \left. - \frac{2}{9}\left[1 + \frac{\Delta^2}{{k^0}^2}\right]\left[a_1^2 + a_1c_3 + \frac{1}{4}c_3^2\right]\right] i(\mathbf{k}\times\mathbf{k}')^3 + \mathcal{O}\left[\frac{\Delta^3}{{k^0}^3}\right] \\
&= f^2k^0\mathcal{A}_{\text{tree}}(n + \pi^0 \to n + \pi^0),
\end{aligned} \tag{4.28}
$$

$$
\begin{aligned}
&\sqrt{2}f^2k^0\mathcal{A}_{\text{tree}}(p + \pi^- \to n + \pi^0) \\
&= \left[-\frac{25}{36}a_1^2 - \frac{5}{18}a_1b_2 - \frac{25}{54}a_1b_3 - \frac{1}{36}b_2^2 - \frac{5}{54}b_2b_3 \right. \qquad (4.29) \\
&\quad \left. - \frac{25}{324}b_3^2 + \frac{4}{9}\left[1 + \frac{\Delta^2}{{k^0}^2}\right]\left[a_1^2 + a_1c_3 + \frac{1}{4}c_3^2\right]\right]\mathbf{k}\cdot\mathbf{k}' \\
&\quad + \frac{2}{9}\frac{\Delta}{k^0}\left[a_1^2 + a_1c_3 + \frac{1}{4}c_3^2\right] i(\mathbf{k}\times\mathbf{k}')^3 + \mathcal{O}\left[\frac{\Delta^3}{{k^0}^3}\right] \\
&= \sqrt{2}f^2k^0\mathcal{A}_{\text{tree}}(n + \pi^+ \to p + \pi^0).
\end{aligned} \tag{4.30}
$$

The past results can be written in terms of the $SU(3)$ chiral invariant couplings D, F, C and H from HBChPT [21]. The relations between the $1/N_c$ coefficients that appear on (3.14) as [20]

$$
D = \frac{1}{2}a_1 + \frac{1}{6}b_3, \tag{4.31a}
$$

$$
F = \frac{1}{3}a_1 + \frac{1}{6}b_2 + \frac{1}{9}b_3, \tag{4.31b}
$$

$$\mathcal{C} = -a_1 - \frac{1}{2}c_3, \tag{4.31c}$$

$$\mathcal{H} = -\frac{3}{2}a_1 - \frac{3}{2}b_2 - \frac{5}{2}b_3. \tag{4.31d}$$

Simplifying the expressions for amplitudes in terms of the chiral coefficients follows

$$\begin{aligned} f^2k^0\mathcal{A}_{\text{tree}}(p+\pi^+ \to p+\pi^+) &= \left[-\frac{1}{2}(D+F)^2 + \frac{1}{9}\left[-\frac{k^0}{k^0-\Delta} + 3\frac{k^0}{k^0+\Delta}\right]\mathcal{C}^2\right]\mathbf{k}\cdot\mathbf{k}' \\ &\quad + \left[\frac{1}{2}(D+F)^2 - \frac{1}{18}\left[\frac{k^0}{k^0-\Delta} + 3\frac{k^0}{k^0+\Delta}\right]\mathcal{C}^2\right] i(\mathbf{k}\times\mathbf{k}')^3 \\ &= f^2k^0\mathcal{A}_{\text{tree}}(n+\pi^- \to n+\pi^-), \end{aligned} \tag{4.32}$$

$$\begin{aligned} f^2k^0\mathcal{A}_{\text{tree}}(p+\pi^- \to p+\pi^-) &= \left[\frac{1}{2}(D+F)^2 - \frac{1}{9}\left[3\frac{k^0}{k^0-\Delta} - \frac{k^0}{k^0+\Delta}\right]\mathcal{C}^2\right]\mathbf{k}\cdot\mathbf{k}' \\ &\quad + \left[\frac{1}{2}(D+F)^2 - \frac{1}{18}\left[3\frac{k^0}{k^0-\Delta} + \frac{k^0}{k^0+\Delta}\right]\mathcal{C}^2\right] i(\mathbf{k}\times\mathbf{k}')^3 \\ &= f^2k^0\mathcal{A}_{\text{tree}}(n+\pi^+ \to n+\pi^+), \end{aligned} \tag{4.33}$$

$$\begin{aligned} f^2k^0\mathcal{A}_{\text{tree}}(p+\pi^0 \to p+\pi^0) &= -\frac{2}{9}\left[\frac{k^0}{k^0-\Delta} - \frac{k^0}{k^0+\Delta}\right]\mathcal{C}^2\mathbf{k}\cdot\mathbf{k}' \\ &\quad + \left[\frac{1}{2}(D+F)^2 - \frac{1}{9}\left[\frac{k^0}{k^0-\Delta} + \frac{k^0}{k^0+\Delta}\right]\mathcal{C}^2\right] i(\mathbf{k}\times\mathbf{k}')^3 \\ &= f^2k^0\mathcal{A}_{\text{tree}}(n+\pi^0 \to n+\pi^0), \end{aligned} \tag{4.34}$$

$$\begin{aligned} \sqrt{2}f^2k^0\mathcal{A}_{\text{tree}}(p+\pi^- \to n+\pi^0) &= \left[-(D+F)^2 + \frac{2}{9}\left[\frac{k^0}{k^0-\Delta} + \frac{k^0}{k^0+\Delta}\right]\mathcal{C}^2\right]\mathbf{k}\cdot\mathbf{k}' \\ &\quad + \frac{1}{9}\left[\frac{k^0}{k^0-\Delta} - \frac{k^0}{k^0+\Delta}\right]\mathcal{C}^2 i(\mathbf{k}\times\mathbf{k}')^3 \\ &= \sqrt{2}f^2k^0\mathcal{A}_{\text{tree}}(n+\pi^+ \to p+\pi^0), \end{aligned} \tag{4.35}$$

that are valid to order $\mathcal{O}(\Delta^3/{k^0}^3)$. These expressions are consistent with HBChPT because F and D come along $BB\pi$ vertices, where $g_A = D+F$ is the axial coupling for neutron beta decay, while $\mathcal{C}$ appear for $BB\pi$ vertices. Additionally, taking the limit $\Delta \to 0$, coefficients of the $\mathcal{C}$ terms do not vanish.

As for the $\mathcal{A}^{(1/2)}_{\rm tree}$ and $\mathcal{A}^{(3/2)}_{\rm tree}$ amplitudes, they are found to be

$$\begin{aligned} f^2 k^0 \mathcal{A}^{(1/2)}_{\rm tree} \\ = \Bigg[\frac{25}{36}a_1^2 + \frac{5}{18}a_1 b_2 + \frac{25}{54}a_1 b_3 + \frac{1}{36}b_2^2 + \frac{5}{54}b_2 b_3 + \frac{25}{324}b_3^2 \\ -\frac{4}{9}\left[1 + \frac{\Delta}{k^0} + \frac{\Delta^2}{k^{0\,2}}\right]\left[a_1^2 + a_1 c_3 + \frac{1}{4}c_3^2\right]\Bigg] \mathbf{k}\cdot\mathbf{k}' \\ + \Bigg[\frac{25}{72}a_1^2 + \frac{5}{36}a_1 b_2 + \frac{25}{108}a_1 b_3 + \frac{1}{72}b_2^2 + \frac{5}{108}b_2 b_3 + \frac{25}{648}b_3^2 \\ -\frac{2}{9}\left[1 + \frac{\Delta}{k^0} + \frac{\Delta^2}{k^{0\,2}}\right]\left[a_1^2 + a_1 c_3 + \frac{1}{4}c_3^2\right]\Bigg] i(\mathbf{k}\times\mathbf{k}')^3 + \mathcal{O}\left[\frac{\Delta^3}{k^{0\,3}}\right], \end{aligned} \tag{4.36}$$

and

$$\begin{aligned} f^2 k^0 \mathcal{A}^{(3/2)}_{\rm tree} \\ = \Bigg[-\frac{25}{72}a_1^2 - \frac{5}{36}a_1 b_2 - \frac{25}{108}a_1 b_3 - \frac{1}{72}b_2^2 - \frac{5}{108}b_2 b_3 - \frac{25}{648}b_3^2 \\ +\frac{2}{9}\left[1 - \frac{2\Delta}{k^0} + \frac{\Delta^2}{k^{0\,2}}\right]\left[a_1^2 + a_1 c_3 + \frac{1}{4}c_3^2\right]\Bigg] \mathbf{k}\cdot\mathbf{k}' \\ + \Bigg[\frac{25}{72}a_1^2 + \frac{5}{36}a_1 b_2 + \frac{25}{108}a_1 b_3 + \frac{1}{72}b_2^2 + \frac{5}{108}b_2 b_3 + \frac{25}{648}b_3^2 \\ -\frac{2}{9}\left[1 - \frac{1}{2}\frac{\Delta}{k^0} + \frac{\Delta^2}{k^{0\,2}}\right]\left[a_1^2 + a_1 c_3 + \frac{1}{4}c_3^2\right]\Bigg] i(\mathbf{k}\times\mathbf{k}')^3 + \mathcal{O}\left[\frac{\Delta^3}{k^{0\,3}}\right], \end{aligned} \tag{4.37}$$

or equivalently,

$$f^2 k^0 \mathcal{A}^{(1/2)}_{\rm tree} = \left[(D+F)^2 - \frac{4}{9}\frac{k^0}{k^0-\Delta}\mathcal{C}^2\right]\mathbf{k}\cdot\mathbf{k}' + \left[\frac{1}{2}(D+F)^2 - \frac{2}{9}\frac{k^0}{k^0-\Delta}\mathcal{C}^2\right] i(\mathbf{k}\times\mathbf{k}')^3, \tag{4.38}$$

and

$$\begin{aligned} f^2 k^0 \mathcal{A}^{(3/2)}_{\rm tree} \quad = \quad & \left[-\frac{1}{2}(D+F)^2 + \frac{1}{9}\left[-\frac{k^0}{k^0-\Delta} + 3\frac{k^0}{k^0+\Delta}\right]\mathcal{C}^2\right]\mathbf{k}\cdot\mathbf{k}' \\ & + \left[\frac{1}{2}(D+F)^2 - \frac{1}{18}\left[\frac{k^0}{k^0-\Delta} + 3\frac{k^0}{k^0+\Delta}\right]\mathcal{C}^2\right] i(\mathbf{k}\times\mathbf{k}')^3, \end{aligned} \tag{4.39}$$

which are valid to order $\mathcal{O}(\Delta^3/{k^0}^3)$.

Isospin relations

The resulting amplitudes for the nucleon-pion scattering at tree-level satisfy some interesting isospin relations between different processes listed below

$$\mathcal{A}_{\text{tree}}(p+\pi^- \to p+\pi^-) - \mathcal{A}_{\text{tree}}(p+\pi^0 \to p+\pi^0) + \frac{1}{\sqrt{2}}\mathcal{A}_{\text{tree}}(p+\pi^- \to n+\pi^0) = 0,$$

$$\mathcal{A}_{\text{tree}}(p+\pi^+ \to p+\pi^+) - \mathcal{A}_{\text{tree}}(p+\pi^- \to p+\pi^-) - \sqrt{2}\mathcal{A}_{\text{tree}}(p+\pi^- \to n+\pi^0) = 0,$$

$$\mathcal{A}_{\text{tree}}(p+\pi^+ \to p+\pi^+) + \mathcal{A}_{\text{tree}}(p+\pi^- \to p+\pi^-) - 2\mathcal{A}_{\text{tree}}(p+\pi^0 \to p+\pi^0) = 0,$$

$$\mathcal{A}_{\text{tree}}(n+\pi^- \to n+\pi^-) - \mathcal{A}_{\text{tree}}(n+\pi^0 \to n+\pi^0) - \frac{1}{\sqrt{2}}\mathcal{A}_{\text{tree}}(n+\pi^+ \to p+\pi^0) = 0,$$

$$\mathcal{A}_{\text{tree}}(n+\pi^+ \to n+\pi^+) - \mathcal{A}_{\text{tree}}(n+\pi^- \to n+\pi^-) + \sqrt{2}\mathcal{A}_{\text{tree}}(n+\pi^+ \to p+\pi^0) = 0,$$

$$\mathcal{A}_{\text{tree}}(n+\pi^+ \to n+\pi^+) + \mathcal{A}_{\text{tree}}(n+\pi^- \to n+\pi^-) - 2\mathcal{A}_{\text{tree}}(n+\pi^0 \to n+\pi^0) = 0.$$

4.2.2 Tree-level scattering amplitude for Fig. 4.1 (c)

The computation of the amplitudes for the contribution of the diagram (c) follows the same process by using (4.12) with the operator A^{ab}_{vertex} from (4.14). So the amplitudes are

$$\begin{aligned}\mathcal{A}_{\text{vertex}}(p+\pi^+ \to p+\pi^+) &= -\frac{i}{4}\frac{k^0}{f^2} \\ &= \mathcal{A}_{\text{vertex}}(n+\pi^- \to n+\pi^-),\end{aligned} \tag{4.40}$$

$$\begin{aligned}\mathcal{A}_{\text{vertex}}(p+\pi^- \to p+\pi^-) &= \frac{i}{4}\frac{k^0}{f^2} \\ &= \mathcal{A}_{\text{vertex}}(n+\pi^+ \to n+\pi^+),\end{aligned} \tag{4.41}$$

$$\begin{aligned}\mathcal{A}_{\text{vertex}}(p+\pi^0 \to p+\pi^0) &= 0 \\ &= \mathcal{A}_{\text{vertex}}(n+\pi^0 \to n+\pi^0),\end{aligned} \tag{4.42}$$

$$\begin{aligned}\mathcal{A}_{\text{vertex}}(p+\pi^- \to n+\pi^0) &= -\frac{i}{2\sqrt{2}}\frac{k^0}{f^2} \\ &= \mathcal{A}_{\text{vertex}}(n+\pi^+ \to p+\pi^0).\end{aligned} \tag{4.43}$$

Isospin relations

Similarly, there are some isospin relations as in the previous case, which relate the amplitudes as

$$\mathcal{A}_{\text{vertex}}(p+\pi^- \to p+\pi^-) - \mathcal{A}_{\text{vertex}}(p+\pi^0 \to p+\pi^0) + \frac{1}{\sqrt{2}}\mathcal{A}_{\text{vertex}}(p+\pi^- \to n+\pi^0) = 0,$$

$$\mathcal{A}_{\text{vertex}}(p+\pi^+ \to p+\pi^+) - \mathcal{A}_{\text{vertex}}(p+\pi^- \to p+\pi^-) - \sqrt{2}\mathcal{A}_{\text{vertex}}(p+\pi^- \to n+\pi^0) = 0,$$

$$\mathcal{A}_{\text{vertex}}(p+\pi^+ \to p+\pi^+) + \mathcal{A}_{\text{vertex}}(p+\pi^- \to p+\pi^-) - 2\mathcal{A}_{\text{vertex}}(p+\pi^0 \to n+\pi^0) = 0,$$

$$\mathcal{A}_{\text{vertex}}(n+\pi^- \to n+\pi^-) - \mathcal{A}_{\text{vertex}}(n+\pi^0 \to n+\pi^0) - \frac{1}{\sqrt{2}}\mathcal{A}_{\text{vertex}}(n+\pi^+ \to p+\pi^0) = 0,$$

$$\mathcal{A}_{\text{vertex}}(n+\pi^+ \to n+\pi^+) - \mathcal{A}_{\text{vertex}}(n+\pi^- \to n+\pi^-) + \sqrt{2}\mathcal{A}_{\text{vertex}}(n+\pi^+ \to p+\pi^0) = 0,$$

$$\mathcal{A}_{\text{vertex}}(n+\pi^+ \to n+\pi^+) + \mathcal{A}_{\text{vertex}}(n+\pi^- \to n+\pi^-) - 2\mathcal{A}_{\text{vertex}}(n+\pi^0 \to n+\pi^0) = 0.$$

4.3 First-order flavor symmetry breaking in the scattering amplitude

The total scattering amplitude obtained from the diagrams in the past section is given in the exact $SU(3)$ symmetry limit. However, this is still a rough approximation to the actual results, which should contain contributions from the effects of perturbative symmetry breaking (SB) of flavor $SU(3)$ symmetry. SB emerges on QCD from the light quark masses and transforms as a flavor adjoint.

In this section, SB contributions are discussed for two different cases: Fig. 4.1 (a,b) and Fig. 4.1 (c). Because each case involves different operator structures.

4.3.1 SB effects of the scattering amplitude for Fig. 4.1 (a,b)

First-order symmetry breaking (SB) effects for scattering amplitude are computed from the tensor product of the operator itself, which transforms under the spin flavor symmetry $SU(2) \times SU(3)$ as $(2, \mathbf{8} \otimes \mathbf{8})$, and the perturbation, that transforms as $(0, \mathbf{8})$. The tensor product of three adjoint representations follows 3.18.

So, effects of SB can be evaluated by constructing the $1/N_c$ expansion for the components of the scattering amplitude, which transform under $SU(2) \times SU(3)$ representations $(2, \mathbf{1})$, $(2, \mathbf{8})$, $(2, \mathbf{10} \otimes \bar{\mathbf{10}})$, $(2, \mathbf{27})$, $(2, \mathbf{35} \otimes \bar{\mathbf{35}})$ and $(2, \mathbf{64})$. As in the past examples, $1/N_c$ expansions have to be expressed in terms of a complete basis of linearly independent operators $\{R^{(ij)(a_1a_2a_3)}\}$, where the general operator $R^{(ij)(a_1a_2a_3)}$ a spin-2 object with three adjoint indices. First-order SB can be considered by setting one of the flavor indices to 8, in this case, $a_3 = 8$. The operator basis contains 170 operators listed in Appendix C.

Since the operators for SB contain three free indices, it is necessary to apply the projector technique from Section 3.4, where the projectors are given as in the equation (3.19). Also, due to the complication of computations at this level, the numerical method to consider projectors as matrices is applied as in the next examples.

The operator $\{T^a, \{T^b, T^c\}\}$ contributes to the scattering amplitude of the process $n + \pi^+ \to n + \pi^+$ through the components with flavor indices $a = (1 - i2)/\sqrt{2}$, $b = (1 - i2)/\sqrt{2}$, and $c = 8$. Using the matrix method, the (118) component of the flavor $\mathbf{8}$ piece becomes,

$$
\begin{aligned}
&[\mathcal{P}^{(\mathbf{8})}]^{118cde}\{T^c, \{T^d, T^e\}\} \\
&= \frac{1}{15}T^1T^1T^8 + \frac{1}{30\sqrt{3}}T^1T^4T^6 + \frac{1}{30\sqrt{3}}T^1T^5T^7 + \frac{1}{30\sqrt{3}}T^1T^6T^4 + \frac{1}{30\sqrt{3}}T^1T^7T^5 \\
&\quad + \frac{4}{15}T^1T^8T^1 + \frac{1}{15}T^2T^2T^8 - \frac{1}{30\sqrt{3}}T^2T^4T^7 + \frac{1}{30\sqrt{3}}T^2T^5T^6 + \frac{1}{30\sqrt{3}}T^2T^6T^5 \\
&\quad - \frac{1}{30\sqrt{3}}T^2T^7T^4 + \frac{4}{15}T^2T^8T^2 + \frac{1}{15}T^3T^3T^8 + \frac{1}{30\sqrt{3}}T^3T^4T^4 + \frac{1}{30\sqrt{3}}T^3T^5T^5
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1}{30\sqrt{3}}T^3T^6T^6-\frac{1}{30\sqrt{3}}T^3T^7T^7+\frac{4}{15}T^3T^8T^3-\frac{1}{15\sqrt{3}}T^4T^1T^6+\frac{1}{15\sqrt{3}}T^4T^2T^7\\
&-\frac{1}{15\sqrt{3}}T^4T^3T^4+\frac{1}{30\sqrt{3}}T^4T^4T^3+\frac{1}{10}T^4T^4T^8+\frac{1}{30\sqrt{3}}T^4T^6T^1-\frac{1}{30\sqrt{3}}T^4T^7T^2\\
&+\frac{1}{5}T^4T^8T^4-\frac{1}{15\sqrt{3}}T^5T^1T^7-\frac{1}{15\sqrt{3}}T^5T^2T^6-\frac{1}{15\sqrt{3}}T^5T^3T^5+\frac{1}{30\sqrt{3}}T^5T^5T^3\\
&+\frac{1}{10}T^5T^5T^8+\frac{1}{30\sqrt{3}}T^5T^6T^2+\frac{1}{30\sqrt{3}}T^5T^7T^1+\frac{1}{5}T^5T^8T^5-\frac{1}{15\sqrt{3}}T^6T^1T^4\\
&-\frac{1}{15\sqrt{3}}T^6T^2T^5+\frac{1}{15\sqrt{3}}T^6T^3T^6+\frac{1}{30\sqrt{3}}T^6T^4T^1+\frac{1}{30\sqrt{3}}T^6T^5T^2-\frac{1}{30\sqrt{3}}T^6T^6T^3\\
&+\frac{1}{10}T^6T^6T^8+\frac{1}{5}T^6T^8T^6-\frac{1}{15\sqrt{3}}T^7T^1T^5+\frac{1}{15\sqrt{3}}T^7T^2T^4+\frac{1}{15\sqrt{3}}T^7T^3T^7\\
&-\frac{1}{30\sqrt{3}}T^7T^4T^2+\frac{1}{30\sqrt{3}}T^7T^5T^1-\frac{1}{30\sqrt{3}}T^7T^7T^3+\frac{1}{10}T^7T^7T^8+\frac{1}{5}T^7T^8T^7\\
&+\frac{1}{15}T^8T^1T^1+\frac{1}{15}T^8T^2T^2+\frac{1}{15}T^8T^3T^3+\frac{1}{10}T^8T^4T^4+\frac{1}{10}T^8T^5T^5+\frac{1}{10}T^8T^6T^6\\
&+\frac{1}{10}T^8T^7T^7+\frac{2}{5}T^8T^8T^8.
\end{aligned}
\tag{4.44}
$$

After obtaining the components for the remaining representations, the completeness relation for the projection operators is fulfilled as

$$[\mathcal{P}^{(1)}+\mathcal{P}^{(8)}+\mathcal{P}^{(10+\overline{10})}+\mathcal{P}^{(27)}+\mathcal{P}^{(35+\overline{35})}+\mathcal{P}^{(64)}]^{118cde}\{T^c,\{T^d,T^e\}\}=\{T^1,\{T^1,T^8\}\}, \tag{4.45}$$

and computing the matrix elements of the operator (4.44) is straightforward; therefore,

$$\langle n\pi^+|[\mathcal{P}^{(8)}]^{118cde}\{T^c,\{T^d,T^e\}\}|n\pi^+\rangle=\frac{1}{2}\sqrt{3}, \tag{4.46}$$

and

$$\langle n\pi^+|[\mathcal{P}^{(r)}]^{118cde}\{T^c,\{T^d,T^e\}\}|n\pi^+\rangle=0, \tag{4.47}$$

for $r\neq 8$.

Repeating the procedure for all the flavor indices in $\{T^a,\{T^b,T^c\}\}$, it is possible to obtain all the contributions of this operator to the chosen scattering process. For the previously considered flavor indices, the resulting expression is

$$\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}k^ik'^j\delta^{ij}\langle n\pi^+|[\mathcal{P}^{(8)}]^{(1-i2)(1-i2)8cde}\{T^c,\{T^d,T^e\}\}|n\pi^+\rangle=\frac{1}{2}\sqrt{3}\mathbf{k}\cdot\mathbf{k}', \tag{4.48}$$

and

$$\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}k^i k'^j \delta^{ij} \left[\langle n\pi^+|[P^{(m)}]^{(1-i2)(1-i2)8cde}\right] \{T^c,\{T^d,T^e\}\}|n\pi^+\rangle = 0, \tag{4.49}$$

for $r \neq 8$.

Applying the process presented above for any baryons and mesons, and using all the operators in the basis, the first-order SB contribution to the scattering amplitude, now denoted as $\delta\mathcal{A}$, can be organized as

$$\begin{aligned}
&f^2 k^0 \delta\mathcal{A}(B+\pi^a \to B'+\pi^b) = \\
&\sum_m \Big[N_c g_1^{(m)} k^i k'^j \langle B'\pi^b|[\mathcal{P}^{(m)}R_1^{(ij)}]^{(ab8)}|B\pi^a\rangle + N_c g_2^{(m)} k^i k'^j \langle B'\pi^b|[\mathcal{P}^{(m)}R_2^{(ij)}]^{(ab8)}|B\pi^a\rangle \\
&\quad + \sum_{r=3}^{16} g_r^{(m)} k^i k'^j \langle B'\pi^b|[\mathcal{P}^{(m)}R_r^{(ij)}]^{(ab8)}|B\pi^a\rangle + \frac{1}{N_c}\sum_{r=17}^{71} g_r^{(m)} k^i k'^j \langle B'\pi^b|[\mathcal{P}^{(m)}R_r^{(ij)}]^{(ab8)}|B\pi^a\rangle \\
&\quad + \frac{1}{N_c^2}\sum_{r=72}^{170} g_r^{(m)} k^i k'^j \langle B'\pi^b|[\mathcal{P}^{(m)}R_r^{(ij)}]^{(ab8)}|B\pi^a\rangle\Big],
\end{aligned} \tag{4.50}$$

where $g_r^{(m)}$, $r = 1, ..., 170$, are undetermined coefficients of order one, and the sum over m covers the six irreps listed above.

Despite the large quantity of unknown coefficients, further simplifications can be achieved. For any process, as the one exemplified here, there are some rearrangements for coefficients, which make the obtained expressions simpler.

As an example, the component related to the flavor irrep 1 of $\delta\mathcal{A}$ for the process $n+\pi^+ \to n+\pi^+$, using the matrix elements of the listed in Appendix C, reads

$$\begin{aligned}
&2\sqrt{3}f^2k^0\delta\mathcal{A}^{(1)}(n+\pi^+ \to n+\pi^+) \\
&= \Big[6g_2^{(1)} + \frac{1}{3}g_{18}^{(1)} + \frac{1}{2}g_{20}^{(1)} + \frac{1}{2}g_{52}^{(1)} + \frac{1}{2}g_{53}^{(1)} + \frac{1}{2}g_{54}^{(1)} + \frac{1}{18}g_{95}^{(1)} + \frac{1}{18}g_{96}^{(1)} + \frac{1}{18}g_{97}^{(1)} \\
&\quad + \frac{1}{18}g_{98}^{(1)} + \frac{1}{18}g_{99}^{(1)} + \frac{1}{18}g_{100}^{(1)} + \frac{1}{3}g_{110}^{(1)} + \frac{1}{3}g_{111}^{(1)} + \frac{1}{3}g_{112}^{(1)} + \frac{1}{9}g_{116}^{(1)} + \frac{1}{9}g_{117}^{(1)} + \frac{1}{9}g_{118}^{(1)} \\
&\quad + \frac{1}{9}g_{119}^{(1)} + \frac{1}{9}g_{120}^{(1)} + \frac{1}{9}g_{121}^{(1)} + \frac{1}{6}g_{134}^{(1)} + \frac{1}{6}g_{135}^{(1)} + \frac{1}{6}g_{136}^{(1)} + \frac{1}{6}g_{137}^{(1)} + \frac{1}{6}g_{138}^{(1)} + \frac{1}{6}g_{139}^{(1)} \\
&\quad + \frac{1}{6}g_{140}^{(1)} + \frac{1}{6}g_{141}^{(1)} + \frac{1}{6}g_{142}^{(1)} + \frac{1}{6}g_{143}^{(1)} + \frac{1}{6}g_{144}^{(1)} + \frac{1}{6}g_{145}^{(1)} - \frac{1}{18}g_{146}^{(1)} - \frac{1}{18}g_{147}^{(1)} - \frac{1}{18}g_{148}^{(1)} \\
&\quad - \frac{1}{18}g_{149}^{(1)}\Big]\mathbf{k}\cdot\mathbf{k}' + \Big[g_4^{(1)} + \frac{1}{6}g_{63}^{(1)} + \frac{1}{6}g_{64}^{(1)} + \frac{1}{6}g_{65}^{(1)} + \frac{1}{6}g_{66}^{(1)} + \frac{1}{6}g_{67}^{(1)} + \frac{1}{6}g_{68}^{(1)} + \frac{1}{6}g_{73}^{(1)}
\end{aligned}$$

$$+\frac{1}{6}g_{80}^{(1)}+\frac{1}{6}g_{81}^{(1)}+\frac{1}{6}g_{82}^{(1)}+\frac{1}{12}g_{122}^{(1)}+\frac{1}{12}g_{123}^{(1)}+\frac{1}{12}g_{124}^{(1)}+\frac{1}{8}g_{125}^{(1)}+\frac{1}{8}g_{126}^{(1)}+\frac{1}{8}g_{127}^{(1)}$$
$$-\frac{1}{24}g_{128}^{(1)}-\frac{1}{24}g_{129}^{(1)}-\frac{1}{24}g_{130}^{(1)}-\frac{1}{16}g_{150}^{(1)}-\frac{1}{16}g_{151}^{(1)}-\frac{1}{16}g_{152}^{(1)}-\frac{1}{16}g_{153}^{(1)}-\frac{1}{16}g_{154}^{(1)}-\frac{1}{16}g_{155}^{(1)}$$
$$-\frac{1}{16}g_{156}^{(1)}-\frac{1}{16}g_{157}^{(1)}-\frac{1}{16}g_{158}^{(1)}-\frac{1}{16}g_{159}^{(1)}-\frac{1}{16}g_{160}^{(1)}-\frac{1}{16}g_{161}^{(1)}-\frac{1}{16}g_{162}^{(1)}-\frac{1}{16}g_{163}^{(1)}$$
$$-\frac{1}{16}g_{164}^{(1)}+\frac{1}{16}g_{165}^{(1)}+\frac{1}{16}g_{166}^{(1)}+\frac{1}{16}g_{167}^{(1)}+\frac{1}{16}g_{168}^{(1)}+\frac{1}{16}g_{169}^{(1)}+\frac{1}{16}g_{170}^{(1)}\Big]i(\mathbf{k}\times\mathbf{k}')^3. \quad (4.51)$$

The applicability of this kind of expression has some disadvantages. The main one is the impossibility of determining all of the free parameters. For the $N+\pi \to N+\pi$ process, simpler expressions can be derived by grouping linear combinations of coefficients $g_r^{(m)}$ into new coefficients as

$$f^2k^0\delta\mathcal{A}^{(1)}(n+\pi^+ \to n+\pi^+) = d_1^{(1)}\mathbf{k}\cdot\mathbf{k}' + e_1^{(1)}i(\mathbf{k}\times\mathbf{k}')^3. \quad (4.52)$$

where the structure of $d^(1)_1$ and $e^(1)_1$ can be easily found from (4.51).

Therefore, the final expressions considering all the components for first-order SB effects to the scattering amplitude for nucleon-pion process are given by

$$\begin{aligned} f^2k^0\delta\mathbf{A}(p+\pi^+ \to p+\pi^+) &= (d_1^{(1)}+d_1^{(8)}+d_1^{(10+\overline{10})}+d_1^{(27)})\mathbf{k}\cdot\mathbf{k}' \\ &\quad + (e_1^{(1)}+e_1^{(8)}+e_1^{(10+\overline{10})}+e_1^{(27)})i(\mathbf{k}\times\mathbf{k}')^3 \\ &= f^2k^0\delta\mathbf{A}(n+\pi^- \to n+\pi^-), \end{aligned} \quad (4.53)$$

$$\begin{aligned} f^2k^0\delta\mathbf{A}(p+\pi^- \to p+\pi^-) &= (d_1^{(1)}+d_1^{(8)}-d_1^{(10+\overline{10})}-d_1^{(27)}+d_2^{(8)}+d_2^{(27)})\mathbf{k}\cdot\mathbf{k}' \\ &\quad + (e_1^{(1)}+e_1^{(8)}-e_1^{(10+\overline{10})}+e_2^{(8)})i(\mathbf{k}\times\mathbf{k}')^3 \\ &= f^2k^0\delta\mathbf{A}(n+\pi^+ \to n+\pi^+), \end{aligned} \quad (4.54)$$

$$\begin{aligned} f^2k^0\delta\mathbf{A}(p+\pi^0 \to p+\pi^0) &= \frac{1}{2}(2d_1^{(1)}+2d_1^{(8)}+d_2^{(8)}+d_2^{(27)})\mathbf{k}\cdot\mathbf{k}' \\ &\quad + \frac{1}{2}(2e_1^{(1)}+2e_1^{(8)}+e_1^{(27)}+e_2^{(8)})i(\mathbf{k}\times\mathbf{k}')^3 \\ &= f^2k^0\delta\mathbf{A}(n+\pi^0 \to n+\pi^0), \end{aligned} \quad (4.55)$$

$$\begin{aligned}\sqrt{2}f^2k^0\delta\mathbf{A}(p+\pi^-\to n+\pi^0) &= (2d_1^{(10+\overline{10})}+2d_1^{(27)}-d_2^{(8)}-d_2^{(27)})\mathbf{k}\cdot\mathbf{k}' \\ &\quad +(2e_1^{(10+\overline{10})}+e_1^{(27)}-e_2^{(8)})i(\mathbf{k}\times\mathbf{k}')^3 \\ &= \sqrt{2}f^2k^0\delta\mathbf{A}(n+\pi^+\to p+\pi^0), \qquad (4.56)\end{aligned}$$

those expressions contain 11 unknown parameters. It should be remarked that irreps $\mathbf{35}+\bar{\mathbf{35}}$ and $\mathbf{64}$ do not participates in the final expressions.

4.3.2 SB effects of the scattering amplitude for Fig. 4.1 (c)

In a completely analogous way, the contribution of diagram (c) can be obtained. Here, the operator A^{ab}_{vertex} is a spin-0 object and contains two adjoint indices. Now, using the same basis of 170 operators and contracting spin indices with δ^{ij}, just 59 remain. Then, after analyzing the matrix elements, there is only one unknown parameter that is required to parametrize SB effects from diagram (c). The amplitudes including SB correction for this case read

$$\begin{aligned}(\mathcal{A}+\delta\mathcal{A})_{\text{vertex}}(p+\pi^+\to p+\pi^+) &= -\frac{i}{4}\frac{k^0}{f^2}(1-h_1) \\ &= (\mathcal{A}+\delta\mathcal{A})_{\text{vertex}}(n+\pi^-\to n+\pi^-), \qquad (4.57)\end{aligned}$$

$$\begin{aligned}(\mathcal{A}+\delta\mathcal{A})_{\text{vertex}}(p+\pi^-\to p+\pi^-) &= \frac{i}{4}\frac{k^0}{f^2}(1+h_1) \\ &= (\mathcal{A}+\delta\mathcal{A})_{\text{vertex}}(n+\pi^+\to n+\pi^+), \qquad (4.58)\end{aligned}$$

$$\begin{aligned}(\mathcal{A}+\delta\mathcal{A})_{\text{vertex}}(p+\pi^0\to p+\pi^0) &= \frac{i}{4}\frac{k^0}{f^2}h_1 \\ &= (\mathcal{A}+\delta\mathcal{A})_{\text{vertex}}(n+\pi^0\to n+\pi^0), \qquad (4.59)\end{aligned}$$

$$\begin{aligned}(\mathcal{A}+\delta\mathcal{A})_{\text{vertex}}(p+\pi^-\to n+\pi^0) &= -\frac{i}{2\sqrt{2}}\frac{k^0}{f^2} \\ &= (\mathcal{A}+\delta\mathcal{A})_{\text{vertex}}(n+\pi^+\to p+\pi^0), \qquad (4.60)\end{aligned}$$

where h_1 is the new unknown parameter, and it is a combination of **1**, **8**, and **27** operator coefficients.

4.4 S-wave scattering lengths

The results presented so far can be applied to the computation of s-wave scattering lengths. The $N\pi$ forward scattering amplitude for a nucleon at rest can be readily obtained from Eqs. (4.38) and (4.39) at threshold. Following the lines of Ref.[36], the s-wave scattering lengths including the baryon mass splitting and first-order SB can be given by

$$\begin{aligned} a^{(1/2)} &= \frac{1}{4\pi}\frac{m_\pi}{f^2}\left[1+\frac{m_\pi}{M_N}\right]^{-1}\left[(D+F)^2-\frac{4}{9}\left[1+\frac{\Delta}{m_\pi}+\frac{\Delta^2}{m_\pi^2}\right]\mathcal{C}^2\right. \\ &\quad \left. + d_1^{(1)}+d_1^{(8)}+d_1^{(10+\overline{10})}+d_1^{(27)}\right] \\ &= a^+ + 2a^-, \end{aligned} \tag{4.61}$$

and

$$\begin{aligned} a^{(3/2)} &= \frac{1}{4\pi}\frac{m_\pi}{f^2}\left[1+\frac{m_\pi}{M_N}\right]^{-1}\left[-\frac{1}{2}(D+F)^2+\frac{2}{9}\left[1-\frac{2\Delta}{m_\pi}+\frac{\Delta^2}{m_\pi^2}\right]\mathcal{C}^2\right. \\ &\quad \left. + d_1^{(1)}+d_1^{(8)}-2d_1^{(10+\overline{10})}-2d_1^{(27)}+\frac{3}{2}d_2^{(8)}+\frac{3}{2}d_2^{(27)}\right] \\ &= a^+ - a^-, \end{aligned} \tag{4.62}$$

which of course are valid to order $\mathcal{O}(\Delta^3/m_\pi^3)$.

Notice that in the limit $\Delta \to 0$ and removing SB effects,

$$a^{(1/2)} + 2a^{(3/2)} = 0, \tag{4.63}$$

which is a well-known result obtained in the context of current algebra [35]. It is important to remark that relation (4.63) is fulfilled even in the presence of the $\mathcal{C}^2$ term, which accounts for the contribution of decuplet baryons.

The usefulness of Eqs. (4.61) and (4.62) relies entirely on the precise determination of the $SU(3)$ invariants D, F, and $\mathcal{C}$ and of course, the six parameters $d_k^{(m)}$ involved in those equations. A detailed analysis requires additional theoretical expressions for which data are available and would involve processes including strangeness. This task implies a non-negligible effort and will be attempted elsewhere.

Chapter 5

Interacting Boson Model

In past chapters, the applicability of the projector technique has been shown and developed in the context of the $1/N_c$ expansion, where the projectors were applied over spin-flavor tensor operators. However, this construction was possible due to the presence of the spin-flavor $SU(2N_f)$ symmetry. In general, it is possible to construct and apply projection operators similarly if there is a $SU(N)$ symmetry in the theory or model and the operators of the theory transform under irreps of the symmetry group. As a last example, the projector technique is applied in the context of **dynamical symmetries** for a nuclear geometrical model.

The atomic nucleus is a system composed of non-trivial many-body quantum objects, which have collective properties that produce a variety of deformed shapes. When a strongly deformed nucleus rotates, it exhibits characteristic rotational band structures, but they keep a certain regularity.

In order to study these many-body quantum systems, some different methods have arisen. One of the most widely used in the understanding of the collective behavior of nuclei is the **Interacting Boson Model (IBM)**, introduced by Arima and Iachelo [6–8]. The fundamentals for the formulation of the model lie in the dynamical symmetries, that arise when the Hamiltonian of a system can be written in terms of the Casimir operators of a chain of groups or algebras. One of the most relevant applications of the group chains is the construction of bases in which the related Hamiltonian can be diagonalized, or the bases transform as representations of appropriate groups [6].

In its early stage, the model was applied to even-even nuclei, to describe collective properties considering pairs of valence nucleons. The pairs conform to two types of bosons: s and d, with positive parity and angular momenta $L = 0$ and $L = 2$, respectively. The first version of the model, IBM-1, treats nucleons as a single kind of boson, while IBM-2, treats protons and neutrons separately.

Considering the IBM-1, the spectrum generating algebra is $U(6)$ and it contains, in one of its chains, the $SU(3)$ as a subalgebra which generates the rotational spectrum [6, 8]. For this group chain, rotational bands appear within the irreps of $SU(3)$ group. In this way, the IBM-1 Hamiltonian is written as a linear combination of linear and quadratic Casimir operators of all the algebras involved in a given chain. Analogously to the $1/N_c$ expansion, linear and quadratic Casimir operators are usually referred to as 1 and 2-body terms, respectively.

The main purpose of this chapter is to construct n-body contributions of the IBM-1 Hamiltonian, for $n = 3, 4$ and 6, in the exact dynamical symmetry limit, by applying the projector technique. (Also, for symmetry breaking, the same technique is implemented in the PDS approach to construct $n = 4$ terms.)

5.1 IBM-1 in the rotational limit

Before work on the IBM-1, it is necessary to introduce the concept of dynamical symmetries, which appear when a Hamiltonian H of a system can be written in terms of invariant operators as

$$H = \sum_G a_G C_G, \tag{5.1}$$

where C_G are the Casimir operators of a chain of algebras

$$G_{\rm dyn} \supset G_1 \supset G_2 \supset \ldots \supset G_{\rm sym}. \tag{5.2}$$

In this case, the spectrum can be obtained in an analytic way using eigenstates $|\lambda_{\rm dyn}, \lambda_1, \lambda_2, \ldots, \lambda_{\rm sym}\rangle$ and eigenvalues $E(\lambda_{\rm dyn}, \lambda_1, \lambda_2, \ldots, \lambda_{\rm sym})$, where the labels $\lambda_{\rm dyn}, \lambda_1, \lambda_2, \ldots, \lambda_{\rm sym}$ are the quantum numbers that characterize irreps of the algebras in the chain. In (5.2), $G_{\rm dyn}$ stands for the spectrum generating algebra of the systems, and operators related to physical observables can be written in terms of its generators and $G_{\rm sym}$ is the symmetry algebra. Moreover, $G_{\rm dyn}$ is broken and the remaining symmetry is $G_{\rm sym}$, which is the true symmetry of the problem [37].

Since IBM-1 is based on a unitary spectrum unitary algebra $G_{\text{dyn}} = U(6)$ and it contains the orthogonal subalgebra $SO(3)$ related to angular momentum. Thus, the Hamiltonian is expanded in elements of $U(6)$ and consists of Hermitian rotational-scalar interactions, which conserve the total number of bosons, $\hat{N} = \hat{n}_s + \hat{n}_d = s^\dagger s + \sum_m d_m^\dagger d_m$. The model admits three different chain algebras,

$$U(6) \supset \left\{ \begin{array}{c} U(5) \supset SO(5) \\ SU_\pm(3) \\ SO_\pm(6) \supset SO(5) \end{array} \right\} \supset SO(3), \tag{5.3}$$

where the first chain is related to the vibrational $U(5)$ [7] and the third one describes γ-unstable $SO(6)$ limits [38]. However, the present work analyzes the rotational bands, so it is focused on the second chain given by

$$U(6) \supset SU_\pm(3) \supset SO(3), \tag{5.4}$$

where the algebras $SU_+(3)$ and $SU_-(3)$ correspond to prolate and oblate shapes of the nuclei, respectively.

In the context of dynamical symmetry, the Hamiltonian for the chain 5.4 is

$$H^{SU(3)} = c_1 + c_2 C^{(1)}_{U(6)} + c_3 C^{(2)}_{U(6)} + c_4 C^{(2)}_{SO(3)} - c_5 C^{(2)}_{SU_\pm(3)}, \tag{5.5}$$

where $C^{(m)}_G$ is the Casimir operator of degree m related to the symmetry group G. In general, any n-body operator is constructed from the product of n creation and n annihilation operators. For example, a quadratic Casimir operator contains two creation and two annihilation operators, so this is a 2-body operator.

The eigenstates of the Hamiltonian are characterized by the respective quantum numbers for each relevant irrep of each symmetry group as

$$|N, (\lambda_\pm, \mu_\pm), K_\pm, L\rangle, \tag{5.6}$$

where N, (λ_+, μ_+), (λ_-, μ_-) and L are related to $U(6)$, $SU_+(3)$, $SU_-(3)$, and $SO(3)$, respectively, and K_+ and K_- are multiplicity labels [39]. The quantum numbers of $SU_-(3)$

and $SU_+(3)$ can be obtained from each other under the interchange $\lambda \leftrightarrow \mu$, so just one $SU(3)$ is necessary from now on and its quantum number is denoted by (λ, μ). For these new numbers, irreps with $\lambda > \mu$ correspond to prolate shapes, and $\lambda < \mu$ correspond to oblate shapes.

Thus, for a given N, there are several $SU(3)$ irreps with (λ, μ) defined as [40]

$$\mu = 0, 2, 4, \dots \tag{5.7a}$$

$$\lambda = 2N - 6l - 2\mu, \qquad l = 0, 1, \dots, N, \tag{5.7b}$$

and for a given $SU(3)$ irrep there are eigenstates with different L expressed in terms of Elliott's quantum number K [41]

$$K = 0, 2, 4, \dots, \min(\lambda, \mu), \tag{5.8a}$$

$$L = \begin{cases} 0, 2, 4, \dots, \max(\lambda, \mu), & \text{for} \quad K = 0, \\ K, K+1, K+2, \dots, K + \max(\lambda, \mu), & \text{for} \quad K > 0. \end{cases} \tag{5.8b}$$

Now, the Casimir related to the $SO(3)$ is the well-known quadratic angular momentum operator

$$C^{(2)}_{SO(3)} = \mathbf{L}^2, \tag{5.9}$$

which describes a particular rotational band according to the eigenvalue $L(L+1)$, for a fixed (λ, μ). Then, considering the 2 body Hamiltonian from 5.5, the energy difference between the rotational bands is given by the $SU(3)$ quadratic Casimir $C^{(2)}_{SU(3)}$. Moreover, the difference between rotational bands can be described by higher-order n-body $SU(3)$ invariant operators in the IBM Hamiltonian. So, it is necessary to create a general n-body $SU(3)$ Hamiltonian.

From the building blocks of the IBM model, creation and annihilation operators of the s and d boson, it is possible to derive a quadratic Casimir for $SU(3)$ as [8, 42]

$$C^{(2)}_{SU(3)} = T^a T^a, \tag{5.10}$$

where T^a are $SU(3)$ generators and contain a suitable combination of creation and annihilation operators which transforms under $SU(3)$ and whose eigenvalue for a certain irrep

(λ, μ) is [43]

$$\frac{1}{3}\left[\lambda^2 + \mu^2 + \lambda\mu + 3(\lambda+\mu)\right]. \tag{5.11}$$

This eigenvalue describes the energy difference between rotational bands in the 2-body Hamiltonian with the dynamical symmetry of the chain given in 5.4.

5.2 Projectors Technique for Higher-Order $SU(3)$-Invariant Operators in IBM Hamiltonian

In order to obtain more control in the description of the energy difference between rotation bands, n-body terms have to be added to the Hamiltonian, for $3 \leq n \leq 6$. For even interactions ($n = 4, 6$), some generators of $SU(3)$ are contracted and the projector technique is needed. For $n = 3$, a slightly different method will be applied. The case of $n = 5$ requires a different analysis, so those operators will not be treated in this work.

5.2.1 4 body terms

Since T^a are 1-body operators and the generators of $SU(3)$, these operators transform under the 8-dimensional adjoint representation. Then, the most general 2-body operator transforms under the tensor product of two adjoint representations $\mathbf{8} \otimes \mathbf{8}$, which can be decomposed as in Eq. 3.17. So, the decomposition of Q^{ab} in terms of the contributions for each representation is

$$Q^{ab} = \alpha_1 Q^{ab}_{(1)} + \alpha_8 Q^{ab}_{(8)} + \alpha_{10+\overline{10}} Q^{ab}_{(10+\overline{10})} + \alpha_{27} Q^{ab}_{(27)}, \tag{5.12}$$

where each term $Q^{ab}_{(\mathrm{irrep})}$ is obtained applying the projector technique from Chapter 2, and using the projection operators from eqs. (4.6) to (4.10), the components read

$$Q^{ab}_{(1)} = \frac{1}{8}\delta^{ab} T^e T^e, \tag{5.13a}$$

$$Q^{ab}_{(8)} = \frac{3}{5} D^{abcd} T^c T^d + \frac{1}{3} F^{abcd} T^c T^d, \tag{5.13b}$$

$$Q^{ab}_{(10+\overline{10})} = \frac{1}{2}(T^a T^b - T^b T^a) - \frac{1}{3} F^{abcd} T^c T^d, \tag{5.13c}$$

$$Q^{ab}_{(27)} = \frac{5}{12}(T^aT^b + T^bT^a) - \frac{5}{24}\delta^{ab}T^eT^e - \frac{7}{20}D^{abcd}T^cT^d + \frac{1}{4}(D^{acbd} + D^{adbc})T^cT^d. \quad (5.13d)$$

Before continue with the computations, a simplification for the $\mathbf{10}+\bar{\mathbf{10}}$ contribution can be achived

$$\begin{aligned} Q^{ab}_{(10+\overline{10})} &= \frac{1}{2}[T^a, T^b] - \frac{1}{n}F^{abcd}T^cT^d \\ &= \frac{1}{2}[T^a, T^b] - \frac{i}{2}f^{abe}T^e \\ &= 0, \end{aligned} \quad (5.14)$$

so Q^{ab} reduces to

$$Q^{ab} = \alpha_1 Q^{ab}_{(1)} + \alpha_8 Q^{ab}_{(8)} + \alpha_{27} Q^{ab}_{(27)}. \quad (5.15)$$

Now, the 4-body contribution to the Hamiltonian $H^{(4)}$ can easily be constructed by contracting Q^{ab} with itself as

$$H^{(4)} = Q^{ab}Q^{ab}, \quad (5.16)$$

and the Hamiltonian with up to 4-body is given by

$$H = c_1 + c_2 C^{(1)}_{U(6)} + c_3 C^{(2)}_{U(6)} + c_4 C^{(2)}_{SO(3)} - c_5 C^{(2)}_{SU(3)} - H^{(4)}, \quad (5.17)$$

here, the sign of the last two terms has been conveniently chosen to describe the experimental spectra data for the nucleus, in terms of the $SU(3)$ invariants. In the past expression, the structure of $H^{(4)}$ is

$$H^{(4)} = \alpha_1^2 H_1^{(4)} + \alpha_8^2 H_8^{(4)} + \alpha_{27}^2 H_{27}^{(4)}, \quad (5.18)$$

with

$$H_1^{(4)} = \frac{1}{8}\left(T^eT^e\right)^2, \quad (5.19a)$$

$$H_8^{(4)} = \frac{3}{5}D^{abcd}T^aT^bT^cT^d + \frac{1}{3}F^{abcd}T^aT^bT^cT^d, \quad (5.19b)$$

$$H_{27}^{(4)} = \frac{7}{8}\left(T^eT^e\right)^2 + if^{abc}T^aT^bT^c + \frac{3}{4}T^eT^e - \frac{3}{5}D^{abcd}T^aT^bT^cT^d, \quad (5.19c)$$

These expressions can be simplified and rewritten in terms of the quadratic Casimir $C^{(2)}_{SU(3)} =$

T^eT^e. For example using the commutator $[T^a, T^b] = if^{abc}T^c$ and the identity

$$d^{abc}d^{ade}f^{bdf} = \frac{n^2-4}{2n}f^{cef}, \tag{5.20}$$

it is possible to reduce $D^{abcd}T^aT^bT^cT^d = \frac{1}{3}(T^eT^e)^2 + \frac{1}{4}T^eT^e$. Analogously, each term in every component of the Hamiltonian can be reduced in terms of the quadratic Casimir as [42]

$$H_1^{(4)} = \frac{1}{8}\left[C^{(2)}_{SU(3)}\right]^2, \tag{5.21a}$$

$$H_8^{(4)} = \frac{1}{5}\left[C^{(2)}_{SU(3)}\right]^2 - \frac{3}{5}C^{(2)}_{SU(3)}, \tag{5.21b}$$

$$H_{27}^{(4)} = \frac{27}{40}\left[C^{(2)}_{SU(3)}\right]^2 - \frac{9}{10}C^{(2)}_{SU(3)}. \tag{5.21c}$$

Thus, the final structure of the Hamiltonian considering 4 body contributions is

$$\begin{aligned} H &= c_1 + c_2C^{(1)}_{U(6)} + c_3C^{(2)}_{U(6)} + c_4C^{(2)}_{SO(3)} - c_5C^{(2)}_{SU(3)} - \frac{\alpha_1^2}{64}\left[C^{(2)}_{SU(3)}\right]^2 \\ &\quad - \frac{\alpha_8^2}{5}\left(\left[C^{(2)}_{SU(3)}\right]^2 - 3\,C^{(2)}_{SU(3)}\right) - \frac{\alpha_{27}^2}{10}\left(\frac{27}{4}\left[C^{(2)}_{SU(3)}\right]^2 - 9\,C^{(2)}_{SU(3)}\right). \end{aligned} \tag{5.22}$$

5.2.2 6-body terms

From the construction of $H^{(4)}$, the contribution to the IBM Hamiltonian of 6-body terms can be straightforwardly constructed. In this case, the tensor product of three adjoint representations is considered, and its decomposition follows 3.18. Thus projection operators have the structure given in 3.19, which act over a general operator Q^{abc}. So, the decomposition for the operator follows

$$Q^{abc} = \alpha_1Q^{abc}_{(1)} + \alpha_8Q^{abc}_{(8)} + \alpha_{10+\overline{10}}Q^{abc}_{(10+\overline{10})} + \alpha_{27}Q^{abc}_{(27)} + \alpha_{35+\overline{35}}Q^{abc}_{(35+\overline{35})} + \alpha_{64}Q^{abc}_{(64)}, \tag{5.23}$$

where

$$Q^{abc}_{(m)} = \mathcal{P}^{(m)}T^aT^bT^c. \tag{5.24}$$

Then, the contribution to the Hamiltonian is

$$H^{(6)} = Q^{abc}Q^{abc}$$

$$= \alpha_1^2 H_1^{(6)} + \alpha_1^8 H_8^{(6)} + \alpha_{10+\overline{10}}^2 H_{10+\overline{10}}^{(6)} + \alpha_{27}^2 H_{27}^{(6)} + \alpha_{35+\overline{35}}^2 H_{35+\overline{35}}^{(6)} + \alpha_{64}^2 H_{64}^{(6)}, \tag{5.25}$$

where

$$H_m^{(6)} = \left[\mathcal{P}^{(m)}\right]^{a'b'c'abc} T^{a'}T^{b'}T^{c'}T^aT^bT^c. \tag{5.26}$$

Since the projection operators $\mathcal{P}^{(m)}$ has a complex analytical structure as has been described in Chapter 2, the explicit expressions for the projectors are not available. However, following the reduction method from [42], the structure of the components of $H^{(6)}$ can be obtained in terms of the Casimir operators as

$$H_1^{(6)} = -\frac{3}{32}\left[C_{SU(3)}^{(2)}\right]^2 + \frac{3}{40}\left[C_{SU(3)}^{(3)}\right]^2, \tag{5.27a}$$

$$H_8^{(6)} = \frac{3}{5}C_{SU(3)}^{(2)} - \frac{3}{5}\left[C_{SU(3)}^{(2)}\right]^2 + \frac{3}{10}\left[C_{SU(3)}^{(2)}\right]^3, \tag{5.27b}$$

$$H_{10+\overline{10}}^{(6)} = \frac{1}{8}\left[C_{SU(3)}^{(2)}\right]^2 + \frac{1}{6}\left[C_{SU(3)}^{(2)}\right]^3 - \frac{1}{2}\left[C_{SU(3)}^{(3)}\right]^2, \tag{5.27c}$$

$$H_{27}^{(6)} = \frac{153}{70}C_{SU(3)}^{(2)} - \frac{2403}{1120}\left[C_{SU(3)}^{(2)}\right]^2 + \frac{9}{70}\left[C_{SU(3)}^{(2)}\right]^3 + \frac{27}{56}\left[C_{SU(3)}^{(3)}\right]^2, \tag{5.27d}$$

$$H_{35+\overline{35}}^{(6)} = 0, \tag{5.27e}$$

$$H_{64}^{(6)} = \frac{12}{7}C_{SU(3)}^{(2)} - \frac{25}{14}\left[C_{SU(3)}^{(2)}\right]^2 + \frac{17}{42}\left[C_{SU(3)}^{(2)}\right]^3 - \frac{2}{35}\left[C_{SU(3)}^{(3)}\right]^2. \tag{5.27f}$$

Notice that $H_m^{(6)}$ terms depend on powers of quadratic and cubic $SU(3)$ Casimir operators.

5.2.3 3-body terms

$SU(3)$-invariant 3-body terms in the IBM Hamiltonian can be constructed using a modified version of the method outlined in previous sections. For the decomposition of $\mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8}$, there are two $SU(3)$-invariant components in the product $T^aT^bT^c$. To isolate these components, the following operators are utilized [42]:

$$\left[\mathcal{P}_A^{(1)}\right]^{a_1a_2b_1b_2} = \frac{1}{3}F^{a_1a_2b_1b_2}, \tag{5.28}$$

$$\left[\mathcal{P}_S^{(1)}\right]^{a_1a_2b_1b_2} = \frac{3}{5}D^{a_1a_2b_1b_2}. \tag{5.29}$$

These operators decompose the antisymmetric and symmetric 8 contributions of the product T^aT^b, denoted as $(T^aT^b)_{8A}$ and $(T^aT^b)_{8S}$, respectively.

If T^a_A denotes the $SU(3)$ generators for the space spanned by the operators T^a, then the generators T^a_{3A} for the space spanned by the tensor product $(T^aT^b)_{8A/S}T^c$ are given by

$$T^a_{3A} = \mathcal{P}^{(1)}_{A/S} T^a_{2A} \otimes \mathbb{1} + \mathcal{P}^{(1)}_{A/S} \otimes T^a_A, \tag{5.30}$$

where

$$T^a_{2A} = T^a_A \otimes \mathbb{1} + \mathbb{1} \otimes T^a_A, \tag{5.31}$$

with components

$$[T^a_A]^{cb} = -if^{acb}. \tag{5.32}$$

Thus, from Eq. (5.30), the quadratic Casimir operator $C = T^a_{3A}T^a_{3A}$ can be expressed as

$$C = \mathcal{P}^{(1)}_{A/S} T^a_{2A} T^a_{2A} \otimes \mathbb{1} + 2\mathcal{P}^{(1)}_{A/S} T^a_{2A} \otimes T^a_A + \mathcal{P}^{(1)}_{A/S} \otimes T^a_A T^a_A, \tag{5.33}$$

where, by Schur's lemma,

$$T^a_A T^a_A = 3\mathbb{1}, \tag{5.34}$$

and

$$T2A^a T^a_{2A} = 6\mathbb{1} \otimes \mathbb{1} + 2T^e_A \otimes T^e_A. \tag{5.35}$$

Using the identities from Ref. [42], we obtain

$$\mathcal{P}^{(1)}_{A/S} T^a_{2A} T^a_{2A} = 3\mathcal{P}^{(1)}_{A/S}. \tag{5.36}$$

Combining these results, the Casimir operator in Eq. (5.33) simplifies to

$$C = 6\mathcal{P}^{(1)}_{A/S} \otimes \mathbb{1} + 2G, \tag{5.37}$$

where

$$G = \mathcal{P}^{(1)}_{A/S} H, \tag{5.38a}$$

$$H = T^a_{2A} \otimes T^a_A, \tag{5.38b}$$

with components

$$[H]^{a_1a_2a_3b_1b_2b_3} = -\left(F^{a_1b_1a_3b_3}\delta^{a_2b_2} + F^{a_2b_2a_3b_3}\delta^{a_1b_1}\right), \tag{5.39a}$$

$$[G]^{a_1a_2a_3b_1b_2b_3} = \left[\mathcal{P}^{(1)}_{A/S}\right]^{a_1a_2b_1'b_2'} [H]^{b_1'b_2'a_3b_1b_2b_3}. \tag{5.39b}$$

The projection operators $\tilde{\mathcal{P}}^{(0)}_{A/S}$ that separate the invariant components of $(T^aT^b)_{8A/S}T^c$ are given by

$$\tilde{\mathcal{P}}^{(0)}_{A/S} = \prod_{i=1}^{3} \frac{C - c_i'\mathcal{P}^{(1)}_{A/S} \otimes \mathbb{1}}{c_0' - c_i'}, \tag{5.40}$$

where the coefficients c_i' represent the eigenvalues of the quadratic Casimir C for the representations 1, 8, $10 \oplus \overline{10}$, and 27 in the tensor product $8 \otimes 8$. These eigenvalues are:

$$c_0' = 0, \quad c_1' = 3, \quad c_2' = 6, \quad c_3' = 8. \tag{5.41}$$

Using Eqs. (5.37) and (5.39) in (5.40), the projection operators $\tilde{\mathcal{P}}^{(0)}_{A/S}$ can be expressed as

$$\left[\tilde{\mathcal{P}}^{(0)}_{A}\right]^{a_1a_2a_3b_1b_2b_3} = \frac{1}{24} f^{a_1a_2a_3} f^{b_1b_2b_3}, \tag{5.42a}$$

$$\left[\tilde{\mathcal{P}}^{(0)}_{S}\right]^{a_1a_2a_3b_1b_2b_3} = \frac{3}{40} d^{a_1a_2a_3} d^{b_1b_2b_3}. \tag{5.42b}$$

Therefore, the two $SU(3)$ invariant contributions of the product of three generators $T^aT^bT^c$ in the IBM Hamiltonian are

$$H^{(3)}_a = d^{abc}T^aT^bT^c, \tag{5.43a}$$

$$H^{(3)}_b = f^{abc}T^aT^bT^c, \tag{5.43b}$$

as expected. The term $H^{(3)}_b$ in (5.43) can be further simplified to

$$H^{(3)}_b = \frac{3}{2} i T^eT^e, \tag{5.44}$$

implying that the only genuine 3-body term that can be included in the IBM Hamiltonian is $H^{(3)}_a$. This term corresponds to the cubic Casimir operator, whose eigenvalues for the irrep (λ, μ) are given by [44]

$$\frac{1}{18}(\lambda-\mu)(2\lambda+\mu+3)(\lambda+2\mu+3). \tag{5.45}$$

5-Body Terms For 5-body terms, as outlined in the previous sections, the invariant components of the tensor $T^aT^bT^cT^dT^e$ are required. This product falls into the representation $8\otimes 8\otimes 8\otimes 8\otimes 8$, which decomposes as

$$\begin{aligned} 8\otimes 8\otimes 8\otimes 8\otimes 8 \;&=\; 32(1)\oplus 145(8)\oplus 100(10\oplus\overline{10})\oplus 180(27)\oplus 20(28\oplus\overline{28}) \\ &\oplus 100(35\oplus\bar{3}5)\oplus 94(64)\oplus 5(80\oplus\overline{80})\oplus 36(81\oplus\overline{81}) \\ &\oplus 20(125)\oplus 4(154\oplus\overline{154})\oplus 216. \end{aligned} \tag{5.46}$$

Thus, there are 32 invariant components in $T^aT^bT^cT^dT^e$, which can be separated through a systematic decomposition of the tensor space $8\otimes 8\otimes 8\otimes 8\otimes 8$, and the corresponding 32 projection operators $\mathcal{P}^{(0)}$ with ten indices. The detailed calculations are overcomplicated out from the illustrative purposes of this section, so they will be discussed in a separated work.

The contributions obtained so far consider the exact dynamical symmetry, but this is an ideal case. The complete analysis for a partial dynamical symmetry is constructed in [42].

Chapter 6

Conclusions

Since the construction of the projection operators method [25], it has shown good applicability over different theories of particle and nuclear physics.

The very first applications appeared on the $1/N_c$ expansion for computations of baryon properties as magnetic moments, quadrupole moments, baryon masses, and axial coupling [18, 24]. Then, the method was implemented using numerical methods for the tree-level analysis of the baryon-meson scattering. The results provided by the application of the projection operators represent an improvement in the classification of n-body operators that appear in the construction of the basis of spin-flavor operators. In this way, the computations can be addressed more systematically, which also implies an improvement in the $1/N_c$ expansion formalism.

Moreover, projection operators were applied in the interacting boson model to construct a higher-order Hamiltonian in the partial symmetries context. This was possible due to the presence of the $SU(N)$ group in the group chain of the rotational bands for atomic nuclei. Additionally, in order to improve the predictive power of the theory, a Hamiltonian with partial dynamical symmetry has been constructed [42].

The above applications are proof of the versatility of the projection operators since the method just needs the theory or model to contain the $SU(N)$ symmetry in the formulation. Also, the obtained results from both cases are evidence of the improvement in the theories generated by using the projectors technique.

In conclusion, the projection operators method represents an improvement in the classification of operators and a systematic way to construct operators that transform under particular irreducible representations, which can be applied in many different theories for particle and nuclear physics.

Appendix A

Identities for SU(N) Structure Constants

The next identities can be derived from the fundamental structure of the $SU(N)$ group

$$f^{abe}f^{cde} + f^{cae}f^{bde} + f^{bce}f^{ade} = 0, \tag{A.1}$$

$$f^{ace}f^{bde} = \frac{2}{N}\left(\delta^{ab}\delta^{cd} - \delta^{ad}\delta^{bc}\right) + d^{abe}d^{cde} - d^{cbe}d^{ade}, \tag{A.2}$$

$$f^{eal}f^{lfc}f^{ebm}f^{mfd} = \frac{1}{2}\left(\delta^{cb}\delta^{da} + \delta^{ca}\delta^{db} + 2\delta^{cd}\delta^{ba}\right) + \frac{N}{4}\left(f^{eab}f^{ecd} + d^{eab}d^{ecd}\right), \tag{A.3}$$

$$f^{hal}f^{elm}f^{fmc}f^{hbn}f^{enp}f^{fpd} = \frac{1}{2}\left(f^{fbc}f^{fad} + f^{fac}f^{fbd}\right) + N\delta^{ab}\delta^{cd} + \frac{N^2}{8}\left(d^{eab}d^{ecd} + f^{eab}f^{ecd}\right), \tag{A.4}$$

$$d^{abc}d^{ade}f^{bdf} = \frac{n^2-4}{2n}f^{cef}, \tag{A.5}$$

$$D^{c_1b_1c_2b_2} + D^{c_2b_1c_1b_2} + D^{c_1c_2b_1b_2} = \frac{1}{3}\left(\delta^{c_2b_1}\delta^{c_1b_2} + \delta^{c_1b_1}\delta^{c_2b_2} + \delta^{c_1c_2}\delta^{b_1b_2}\right). \tag{A.6}$$

Appendix B

Operators Basis at Tree Level

The operators $S_m^{(ij)(ab)}$ that constitute the basis, comprising up to 7-body operators, read

$$S_1^{(ij)(ab)} = i\delta^{ij} f^{abe} T^e,$$
$$S_2^{(ij)(ab)} = i\epsilon^{ijr}\delta^{ab} J^r,$$
$$S_3^{(ij)(ab)} = i\epsilon^{ijr} d^{abe} G^{re},$$
$$S_4^{(ij)(ab)} = \delta^{ab}\{J^i, J^j\},$$
$$S_5^{(ij)(ab)} = \delta^{ij}\delta^{ab} J^2,$$
$$S_6^{(ij)(ab)} = \{G^{ia}, G^{jb}\},$$
$$S_7^{(ij)(ab)} = \{G^{ib}, G^{ja}\},$$
$$S_8^{(ij)(ab)} = \delta^{ij}\{G^{ra}, G^{rb}\},$$
$$S_9^{(ij)(ab)} = i\epsilon^{ijr}\{G^{ra}, T^b\},$$
$$S_{10}^{(ij)(ab)} = i\epsilon^{ijr}\{G^{rb}, T^a\},$$
$$S_{11}^{(ij)(ab)} = d^{abe}\{J^j, G^{ie}\},$$
$$S_{12}^{(ij)(ab)} = if^{abe}\{J^i, G^{je}\},$$
$$S_{13}^{(ij)(ab)} = if^{abe}\{J^j, G^{ie}\},$$
$$S_{14}^{(ij)(ab)} = \delta^{ij} d^{abe}\{J^r, G^{re}\},$$
$$S_{15}^{(ij)(ab)} = \epsilon^{ijr} f^{abe}\mathcal{D}_2^{re},$$
$$S_{16}^{(ij)(ab)} = i\epsilon^{ijr} d^{abe}\mathcal{D}_3^{re},$$
$$S_{17}^{(ij)(ab)} = i\epsilon^{ijr} d^{abe}\mathcal{O}_3^{re},$$
$$S_{18}^{(ij)(ab)} = i\epsilon^{ijr}\{J^r, \{T^a, T^b\}\},$$
$$S_{19}^{(ij)(ab)} = i\epsilon^{ijm}\{J^m, \{G^{ra}, G^{rb}\}\},$$
$$S_{20}^{(ij)(ab)} = if^{abe}\{T^e, \{J^i, J^j\}\},$$
$$S_{21}^{(ij)(ab)} = i\delta^{ij} f^{abe}\{J^2, T^e\},$$
$$S_{22}^{(ij)(ab)} = i\epsilon^{ijr}\delta^{ab}\{J^2, J^r\},$$
$$S_{23}^{(ij)(ab)} = i\epsilon^{imr}\{G^{ja}, \{J^m, G^{rb}\}\},$$
$$S_{24}^{(ij)(ab)} = i\epsilon^{jmr}\{G^{ia}, \{J^m, G^{rb}\}\},$$
$$S_{25}^{(ij)(ab)} = i\epsilon^{imr}\{G^{jb}, \{J^m, G^{ra}\}\},$$
$$S_{26}^{(ij)(ab)} = i\epsilon^{jmr}\{G^{ib}, \{J^m, G^{ra}\}\},$$
$$S_{27}^{(ij)(ab)} = i\epsilon^{ijm}\{G^{ma}, \{J^r, G^{rb}\}\},$$
$$S_{28}^{(ij)(ab)} = i\epsilon^{ijm}\{G^{mb}, \{J^r, G^{ra}\}\},$$
$$S_{29}^{(ij)(ab)} = if^{aeg} d^{beh}\{T^h, \{J^i, G^{jg}\}\},$$
$$S_{30}^{(ij)(ab)} = id^{aeg} f^{beh}\{T^g, \{J^j, G^{ih}\}\},$$
$$S_{31}^{(ij)(ab)} = d^{abe}[J^2, \{J^i, G^{je}\}],$$
$$S_{32}^{(ij)(ab)} = d^{abe}[J^2, \{J^j, G^{ie}\}],$$
$$S_{33}^{(ij)(ab)} = if^{abe}[J^2, \{J^i, G^{je}\}],$$
$$S_{34}^{(ij)(ab)} = if^{abe}[J^2, \{J^j, G^{ie}\}],$$

$$S_{35}^{(ij)(ab)} = i\epsilon^{ijr}[J^2,\{G^{ra},T^b\}],$$
$$S_{36}^{(ij)(ab)} = i\epsilon^{ijr}[J^2,\{G^{rb},T^a\}],$$
$$S_{37}^{(ij)(ab)} = \epsilon^{ijr}f^{abe}\mathcal{D}_4^{re},$$
$$S_{38}^{(ij)(ab)} = if^{abe}\{\{J^i,J^j\},\{J^r,G^{re}\}\},$$
$$S_{39}^{(ij)(ab)} = i\epsilon^{ijm}\{\mathcal{D}_2^{mb},\{J^r,G^{ra}\}\},$$
$$S_{40}^{(ij)(ab)} = i\epsilon^{ijm}\{\mathcal{D}_2^{ma},\{J^r,G^{rb}\}\},$$
$$S_{41}^{(ij)(ab)} = i\epsilon^{ijr}\{J^2,\{G^{ra},T^b\}\},$$
$$S_{42}^{(ij)(ab)} = i\epsilon^{ijr}\{J^2,\{G^{rb},T^a\}\},$$
$$S_{43}^{(ij)(ab)} = if^{abe}\{J^2,\{J^i,G^{je}\}\},$$
$$S_{44}^{(ij)(ab)} = if^{abe}\{J^2,\{J^j,G^{ie}\}\},$$
$$S_{45}^{(ij)(ab)} = \{J^2,\{G^{ia},G^{jb}\}\},$$
$$S_{46}^{(ij)(ab)} = \{J^2,\{G^{ib},G^{ja}\}\},$$
$$S_{47}^{(ij)(ab)} = d^{abe}\{\{J^i,J^j\},\{J^r,G^{re}\}\},$$
$$S_{48}^{(ij)(ab)} = \delta^{ab}\{J^2,\{J^i,J^j\}\},$$
$$S_{49}^{(ij)(ab)} = \epsilon^{ijk}\epsilon^{rml}\{J^k,\{G^{ra},\{J^m,G^{lb}\}\}\},$$
$$S_{50}^{(ij)(ab)} = i\epsilon^{iml}[\{J^j,\{J^m,G^{la}\}\},\{J^r,G^{rb}\}],$$
$$S_{51}^{(ij)(ab)} = i\epsilon^{jml}[\{J^i,\{J^m,G^{la}\}\},\{J^r,G^{rb}\}],$$
$$S_{52}^{(ij)(ab)} = i\epsilon^{jml}[\{J^i,\{J^m,G^{lb}\}\},\{J^r,G^{ra}\}],$$
$$S_{53}^{(ij)(ab)} = i\epsilon^{iml}[\{J^j,\{J^m,G^{lb}\}\},\{J^r,G^{ra}\}],$$
$$S_{54}^{(ij)(ab)} = \{G^{ia},\mathcal{O}_3^{jb}\},$$
$$S_{55}^{(ij)(ab)} = i\epsilon^{ijm}[J^2,\{G^{mb},\{J^r,G^{ra}\}\}],$$
$$S_{56}^{(ij)(ab)} = i\epsilon^{ijm}\{J^2,[G^{mb},\{J^r,G^{ra}\}]\},$$
$$S_{57}^{(ij)(ab)} = \delta^{ij}\{J^2,\{G^{ra},G^{rb}\}\},$$
$$S_{58}^{(ij)(ab)} = \delta^{ij}d^{abe}\{J^2,\{J^r,G^{re}\}\},$$
$$S_{59}^{(ij)(ab)} = \delta^{ij}\delta^{ab}\{J^2,J^2\},$$
$$S_{60}^{(ij)(ab)} = \{[J^2,G^{ia}],[J^2,G^{jb}]\},$$
$$S_{61}^{(ij)(ab)} = i\epsilon^{jmr}[J^2,\{G^{ib},\{J^m,G^{ra}\}\}],$$
$$S_{62}^{(ij)(ab)} = i\epsilon^{ijr}d^{abe}\mathcal{D}_5^{re},$$
$$S_{63}^{(ij)(ab)} = \epsilon^{ijr}f^{abe}\mathcal{O}_5^{re},$$
$$S_{64}^{(ij)(ab)} = i\epsilon^{ijr}d^{abe}\mathcal{O}_5^{re},$$
$$S_{65}^{(ij)(ab)} = \{\mathcal{O}_3^{ia},\mathcal{D}_2^{jb}\},$$
$$S_{66}^{(ij)(ab)} = \{\mathcal{D}_2^{ia},\mathcal{O}_3^{jb}\},$$
$$S_{67}^{(ij)(ab)} = \{J^2,\{T^a,\{J^j,G^{ib}\}\}\},$$
$$S_{68}^{(ij)(ab)} = \{J^2,\{T^b,\{J^i,G^{ja}\}\}\},$$
$$S_{69}^{(ij)(ab)} = if^{abe}\{J^2,\{T^e,\{J^i,J^j\}\}\},$$
$$S_{70}^{(ij)(ab)} = i\delta^{ij}f^{abe}\{J^2,\{J^2,T^e\}\},$$
$$S_{71}^{(ij)(ab)} = i\epsilon^{ijr}\delta^{ab}\{J^2,\{J^2,J^r\}\},$$
$$S_{72}^{(ij)(ab)} = i\epsilon^{ijm}\{J^2,\{J^m,\{G^{ra},G^{rb}\}\}\},$$
$$S_{73}^{(ij)(ab)} = i\epsilon^{imr}\{J^2,\{G^{ja},\{J^m,G^{rb}\}\}\},$$
$$S_{74}^{(ij)(ab)} = i\epsilon^{jmr}\{J^2,\{G^{ia},\{J^m,G^{rb}\}\}\},$$
$$S_{75}^{(ij)(ab)} = i\epsilon^{imr}\{J^2,\{G^{jb},\{J^m,G^{ra}\}\}\},$$
$$S_{76}^{(ij)(ab)} = i\epsilon^{jmr}\{J^2,\{G^{ib},\{J^m,G^{ra}\}\}\},$$
$$S_{77}^{(ij)(ab)} = i\epsilon^{ijm}\{J^2,\{G^{ma},\{J^r,G^{rb}\}\}\},$$
$$S_{78}^{(ij)(ab)} = i\epsilon^{ijm}\{J^2,\{G^{mb},\{J^r,G^{ra}\}\}\},$$
$$S_{79}^{(ij)(ab)} = if^{aeg}d^{beh}\{J^2,\{T^h,\{J^i,G^{jg}\}\}\},$$
$$S_{80}^{(ij)(ab)} = id^{aeg}f^{beh}\{J^2,\{T^g,\{J^j,G^{ih}\}\}\},$$
$$S_{81}^{(ij)(ab)} = d^{abe}\{J^2,[J^2,\{J^i,G^{je}\}]\},$$
$$S_{82}^{(ij)(ab)} = d^{abe}\{J^2,[J^2,\{J^j,G^{ie}\}]\},$$
$$S_{83}^{(ij)(ab)} = i\epsilon^{ijl}\{J^l,\{\{J^r,G^{ra}\},\{J^m,G^{mb}\}\}\},$$
$$S_{84}^{(ij)(ab)} = \{\{J^i,J^j\},\{T^a,\{J^r,G^{rb}\}\}\},$$
$$S_{85}^{(ij)(ab)} = \{\{J^i,J^j\},\{T^b,\{J^r,G^{ra}\}\}\},$$
$$S_{86}^{(ij)(ab)} = i\epsilon^{mlr}\{\{J^i,J^j\},\{G^{mb},\{J^l,G^{ra}\}\}\},$$
$$S_{87}^{(ij)(ab)} = i\epsilon^{jlm}\{\{J^i,\{J^l,G^{ma}\}\},\{J^r,G^{rb}\}\},$$
$$S_{88}^{(ij)(ab)} = i\epsilon^{ilm}\{\{J^j,\{J^l,G^{ma}\}\},\{J^r,G^{rb}\}\},$$
$$S_{89}^{(ij)(ab)} = i\epsilon^{jlm}\{\{J^i,\{J^l,G^{mb}\}\},\{J^r,G^{ra}\}\},$$
$$S_{90}^{(ij)(ab)} = i\epsilon^{ilm}\{\{J^j,\{J^l,G^{mb}\}\},\{J^r,G^{ra}\}\},$$

$$
\begin{aligned}
S_{91}^{(ij)(ab)} &= i\epsilon^{imr}\{G^{jb},\{J^2,\{J^m,G^{ra}\}\}\}, & S_{92}^{(ij)(ab)} &= i\epsilon^{ijr}\{J^2,[J^2,\{G^{ra},T^b\}]\}, \\
S_{93}^{(ij)(ab)} &= i\epsilon^{ijr}\{J^2,[J^2,\{G^{rb},T^a\}]\}, & S_{94}^{(ij)(ab)} &= if^{abe}\{J^2,[J^2,\{J^i,G^{je}\}]\}, \\
S_{95}^{(ij)(ab)} &= if^{abe}\{J^2,[J^2,\{J^j,G^{ie}\}]\}, & S_{96}^{(ij)(ab)} &= \epsilon^{ijr}f^{abe}\mathcal{D}_6^{re}, \\
S_{97}^{(ij)(ab)} &= if^{abe}\{J^2,\{\{J^i,J^j\},\{J^r,G^{re}\}\}\}, & S_{98}^{(ij)(ab)} &= i\epsilon^{ijm}\{J^2,\{\mathcal{D}_2^{mb},\{J^r,G^{ra}\}\}\}, \\
S_{99}^{(ij)(ab)} &= i\epsilon^{ijm}\{J^2,\{\mathcal{D}_2^{ma},\{J^r,G^{rb}\}\}\}, & S_{100}^{(ij)(ab)} &= i\epsilon^{ijr}\{J^2,\{J^2,\{G^{ra},T^b\}\}\}, \\
S_{101}^{(ij)(ab)} &= i\epsilon^{ijr}\{J^2,\{J^2,\{G^{rb},T^a\}\}\}, & S_{102}^{(ij)(ab)} &= if^{abe}\{J^2,\{J^2,\{J^i,G^{je}\}\}\}, \\
S_{103}^{(ij)(ab)} &= if^{abe}\{J^2,\{J^2,\{J^j,G^{ie}\}\}\}, & S_{104}^{(ij)(ab)} &= \{J^2,\{J^2,\{G^{ia},G^{jb}\}\}\}, \\
S_{105}^{(ij)(ab)} &= \{J^2,\{J^2,\{G^{ib},G^{ja}\}\}\}, & S_{106}^{(ij)(ab)} &= d^{abe}\{J^2,\{\{J^i,J^j\},\{J^r,G^{re}\}\}\}, \\
S_{107}^{(ij)(ab)} &= \delta^{ab}\{J^2,\{J^2,\{J^i,J^j\}\}\}, & S_{108}^{(ij)(ab)} &= i\epsilon^{jml}\{J^2,[\{J^i,\{J^m,G^{la}\}\},\{J^r,G^{rb}\}]\}, \\
S_{109}^{(ij)(ab)} &= i\epsilon^{jml}\{J^2,[\{J^i,\{J^m,G^{lb}\}\},\{J^r,G^{ra}\}]\}, & S_{110}^{(ij)(ab)} &= i\epsilon^{iml}\{J^2,[\{J^j,\{J^m,G^{lb}\}\},\{J^r,G^{ra}\}]\}, \\
S_{111}^{(ij)(ab)} &= \{J^2,\{G^{ia},\mathcal{O}_3^{jb}\}\}, & S_{112}^{(ij)(ab)} &= i\epsilon^{ijm}\{J^2,[J^2,\{G^{mb},\{J^r,G^{ra}\}\}]\}, \\
S_{113}^{(ij)(ab)} &= i\epsilon^{ijm}\{J^2,\{J^2,[G^{mb},\{J^r,G^{ra}\}]\}\}, & S_{114}^{(ij)(ab)} &= \delta^{ij}\{J^2,\{J^2,\{G^{ra},G^{rb}\}\}\}, \\
S_{115}^{(ij)(ab)} &= \delta^{ij}d^{abe}\{J^2,\{J^2,\{J^r,G^{re}\}\}\}, & S_{116}^{(ij)(ab)} &= \delta^{ij}\delta^{ab}\{J^2,\{J^2,J^2\}\}, \\
S_{117}^{(ij)(ab)} &= \{J^2,\{[J^2,G^{ia}],[J^2,G^{jb}]\}\}, & S_{118}^{(ij)(ab)} &= i\epsilon^{imr}\{J^2,[J^2,\{G^{jb},\{J^m,G^{ra}\}\}]\}, \\
S_{119}^{(ij)(ab)} &= i\epsilon^{jmr}\{J^2,[J^2,\{G^{ib},\{J^m,G^{ra}\}\}]\}, & S_{120}^{(ij)(ab)} &= i\epsilon^{ijr}d^{abe}\mathcal{D}_7^{re}, \\
S_{121}^{(ij)(ab)} &= i\epsilon^{ijr}d^{abe}\mathcal{O}_7^{re}, & S_{122}^{(ij)(ab)} &= if^{abe}\{J^2,\{J^2,\{T^e,\{J^i,J^j\}\}\}\}, \\
S_{123}^{(ij)(ab)} &= i\delta^{ij}f^{abe}\{J^2,\{J^2,\{J^2,T^e\}\}\}, & S_{124}^{(ij)(ab)} &= i\epsilon^{ijr}\delta^{ab}\{J^2,\{J^2,\{J^2,J^r\}\}\}, \\
S_{125}^{(ij)(ab)} &= i\epsilon^{imr}\{J^2,\{J^2,\{G^{ja},\{J^m,G^{rb}\}\}\}\}, & S_{126}^{(ij)(ab)} &= i\epsilon^{jmr}\{J^2,\{J^2,\{G^{ia},\{J^m,G^{rb}\}\}\}\}, \\
S_{127}^{(ij)(ab)} &= i\epsilon^{imr}\{J^2,\{J^2,\{G^{jb},\{J^m,G^{ra}\}\}\}\}, & S_{128}^{(ij)(ab)} &= i\epsilon^{jmr}\{J^2,\{J^2,\{G^{ib},\{J^m,G^{ra}\}\}\}\}, \\
S_{129}^{(ij)(ab)} &= i\epsilon^{ijm}\{J^2,\{J^2,\{G^{ma},\{J^r,G^{rb}\}\}\}\}, & S_{130}^{(ij)(ab)} &= i\epsilon^{ijm}\{J^2,\{J^2,\{G^{mb},\{J^r,G^{ra}\}\}\}\}, \\
S_{131}^{(ij)(ab)} &= if^{aeg}d^{beh}\{J^2,\{J^2,\{T^h,\{J^i,G^{jg}\}\}\}\}, & S_{132}^{(ij)(ab)} &= id^{aeg}f^{beh}\{J^2,\{J^2,\{T^g,\{J^j,G^{ih}\}\}\}\}, \\
S_{133}^{(ij)(ab)} &= d^{abe}\{J^2,\{J^2,[J^2,\{J^i,G^{je}\}]\}\}, & S_{134}^{(ij)(ab)} &= d^{abe}\{J^2,\{J^2,[J^2,\{J^j,G^{ie}\}]\}\}, \\
S_{135}^{(ij)(ab)} &= i\epsilon^{ijl}\{J^2,\{J^l,\{\{J^r,G^{ra}\},\{J^m,G^{mb}\}\}\}\}, & S_{136}^{(ij)(ab)} &= i\epsilon^{mlr}\{J^2,\{\{J^i,J^j\},\{G^{mb},\{J^l,G^{ra}\}\}\}\}, \\
S_{137}^{(ij)(ab)} &= i\epsilon^{jlm}\{J^2,\{\{J^i,\{J^l,G^{ma}\}\},\{J^r,G^{rb}\}\}\}, & S_{138}^{(ij)(ab)} &= i\epsilon^{jlm}\{J^2,\{\{J^i,\{J^l,G^{mb}\}\},\{J^r,G^{ra}\}\}\}, \\
S_{139}^{(ij)(ab)} &= i\epsilon^{imr}\{J^2,\{G^{jb},\{J^2,\{J^m,G^{ra}\}\}\}\}. & & \qquad\qquad (B.1)
\end{aligned}
$$

The operator coefficients $c_m^{(\mathrm{s})}$ and $c_m^{(\mathrm{a})}$ ($m = 1, \ldots, 139$) that that also appear on the baryon matrix elements in (4.13) are

$$c_1^{(s)} = \frac{1}{4}a_1^2, \tag{B.2}$$

$$c_2^{(s)} = 0, \tag{B.3}$$

$$c_3^{(s)} = 0, \tag{B.4}$$

$$c_4^{(s)} = \frac{\Delta}{k^0}\left[-\frac{1}{18}a_1^2\right], \tag{B.5}$$

$$c_5^{(s)} = \frac{\Delta}{k^0}\left[\frac{1}{9}a_1^2\right], \tag{B.6}$$

$$c_6^{(s)} = \frac{\Delta}{k^0}\left[\frac{1}{6}a_1^2\right], \tag{B.7}$$

$$c_7^{(s)} = \frac{\Delta}{k^0}\left[\frac{1}{6}a_1^2\right], \tag{B.8}$$

$$c_8^{(s)} = \frac{\Delta}{k^0}\left[-\frac{1}{3}a_1^2\right], \tag{B.9}$$

$$c_9^{(s)} = 0, \tag{B.10}$$

$$c_{10}^{(s)} = 0, \tag{B.11}$$

$$c_{11}^{(s)} = \frac{\Delta}{k^0}\left[-\frac{1}{6}a_1^2\right], \tag{B.12}$$

$$c_{12}^{(s)} = \frac{1}{6}a_1b_2 + \frac{1}{9}a_1b_3 - \frac{1}{18}a_1c_3 + \frac{1}{27}b_3^2 - \frac{1}{27}b_3c_3 + \frac{1}{81}c_3^2 + \frac{\Delta}{k^0}\left[\frac{1}{216}a_1c_3 + \frac{613}{11664}b_3c_3 - \frac{1481}{69984}c_3^2\right]$$

$$+\frac{\Delta^2}{k^{0 2}}\left[-\frac{1}{9}a_1^2-\frac{1}{27}a_1b_3+\frac{2}{81}a_1c_3-\frac{149}{17496}c_3^2\right], \tag{B.13}$$

$$\begin{aligned} c_{13}^{(\mathrm{s})} &= \frac{1}{6}a_1b_2+\frac{1}{9}a_1b_3+\frac{1}{81}b_3^2-\frac{1}{18}a_1c_3-\frac{1}{81}b_3c_3+\frac{\Delta}{k^0}\left[-\frac{1}{216}a_1c_3-\frac{613}{11664}b_3c_3+\frac{1733}{69984}c_3^2\right] \\ &+\frac{\Delta^2}{k^{0 2}}\left[-\frac{1}{81}a_1b_3+\frac{23}{4374}c_3^2\right], \end{aligned} \tag{B.14}$$

$$c_{14}^{(\mathrm{s})}=\frac{\Delta}{k^0}\left[\frac{1}{6}a_1^2\right], \tag{B.15}$$

$$c_{15}^{(\mathrm{s})}=\frac{\Delta}{k^0}\left[\frac{1}{9}a_1c_3+\frac{1}{648}b_3c_3+\frac{13}{972}c_3^2\right]+\frac{\Delta^2}{k^{0 2}}\left[\frac{7}{3888}c_3^2\right], \tag{B.16}$$

$$c_{16}^{(\mathrm{s})}=-\frac{2}{81}b_3^2+\frac{2}{81}b_3c_3-\frac{1}{81}c_3^2+\frac{\Delta}{k^0}\left[-\frac{7}{3888}c_3^2\right]+\frac{\Delta^2}{k^{0 2}}\left[\frac{2}{81}a_1b_3-\frac{1}{27}a_1c_3-\frac{13}{11664}c_3^2\right], \tag{B.17}$$

$$\begin{aligned} c_{17}^{(\mathrm{s})} &= -\frac{1}{81}b_3^2+\frac{1}{81}b_3c_3-\frac{1}{162}c_3^2+\frac{\Delta}{k^0}\left[-\frac{1}{36}a_1b_3-\frac{1}{24}a_1c_3-\frac{109}{972}b_3c_3+\frac{431}{7776}c_3^2\right] \\ &+\frac{\Delta^2}{k^{0 2}}\left[\frac{1}{81}a_1b_3-\frac{1}{108}a_1c_3+\frac{485}{23328}c_3^2\right], \end{aligned} \tag{B.18}$$

$$c_{18}^{(\mathrm{s})}=0, \tag{B.19}$$

$$c_{19}^{(\mathrm{s})}=\frac{2}{81}b_3^2-\frac{2}{81}b_3c_3+\frac{1}{81}c_3^2+\frac{\Delta}{k^0}\left[-\frac{1}{36}a_1c_3-\frac{1}{243}b_3c_3+\frac{127}{5832}c_3^2\right]+\frac{\Delta^2}{k^{0 2}}\left[-\frac{2}{81}a_1b_3+\frac{1}{27}a_1c_3+\frac{7}{3888}c_3^2\right], \tag{B.20}$$

$$c_{20}^{(\mathrm{s})}=\frac{1}{36}b_2^2+\frac{1}{54}b_3^2-\frac{1}{54}b_3c_3+\frac{1}{216}c_3^2+\frac{\Delta}{k^0}\left[\frac{7}{2592}c_3^2\right]+\frac{\Delta^2}{k^{0 2}}\left[-\frac{1}{54}a_1b_3+\frac{1}{108}a_1c_3+\frac{13}{7776}c_3^2\right], \tag{B.21}$$

$$c_{21}^{(s)} = \frac{1}{18}a_1c_3 - \frac{2}{81}b_3^2 + \frac{2}{81}b_3c_3 - \frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[-\frac{7}{3888}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[\frac{1}{18}a_1^2 + \frac{2}{81}a_1b_3 - \frac{1}{81}a_1c_3 + \frac{19}{11664}c_3^2\right], \tag{B.22}$$

$$c_{22}^{(s)} = -\frac{4}{243}b_3^2 + \frac{4}{243}b_3c_3 - \frac{2}{243}c_3^2 + \frac{\Delta}{k^0}\left[-\frac{7}{5832}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[\frac{4}{243}a_1b_3 - \frac{2}{81}a_1c_3 - \frac{13}{17496}c_3^2\right], \tag{B.23}$$

$$c_{23}^{(s)} = -\frac{1}{18}a_1c_3 - \frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[-\frac{1}{216}a_1c_3 + \frac{269}{11664}b_3c_3 - \frac{41}{34992}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{36}a_1c_3 - \frac{775}{69984}c_3^2\right], \tag{B.24}$$

$$\begin{aligned} c_{24}^{(s)} &= -\frac{1}{18}a_1c_3 + \frac{4}{81}b_3^2 - \frac{4}{81}b_3c_3 + \frac{1}{54}c_3^2 + \frac{\Delta}{k^0}\left[\frac{1}{216}a_1c_3 - \frac{269}{11664}b_3c_3 + \frac{167}{34992}c_3^2\right] \\ &\quad + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{9}a_1^2 - \frac{4}{81}a_1b_3 + \frac{17}{324}a_1c_3 + \frac{547}{69984}c_3^2\right], \end{aligned} \tag{B.25}$$

$$\begin{aligned} c_{25}^{(s)} &= \frac{1}{18}a_1c_3 - \frac{2}{81}b_3^2 + \frac{2}{81}b_3c_3 - \frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[-\frac{5}{216}a_1c_3 - \frac{317}{11664}b_3c_3 + \frac{677}{34992}c_3^2\right] \\ &\quad + \frac{\Delta^2}{k^{0\,2}}\left[\frac{2}{81}a_1b_3 - \frac{1}{108}a_1c_3 + \frac{745}{69984}c_3^2\right], \end{aligned} \tag{B.26}$$

$$\begin{aligned} c_{26}^{(s)} &= \frac{1}{18}a_1c_3 - \frac{2}{81}b_3^2 + \frac{2}{81}b_3c_3 - \frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[\frac{5}{216}a_1c_3 + \frac{317}{11664}b_3c_3 - \frac{803}{34992}c_3^2\right] \\ &\quad + \frac{\Delta^2}{k^{0\,2}}\left[\frac{1}{9}a_1^2 + \frac{2}{81}a_1b_3 - \frac{5}{324}a_1c_3 - \frac{517}{69984}c_3^2\right], \end{aligned} \tag{B.27}$$

$$c_{27}^{(s)} = \frac{1}{81}b_3^2 - \frac{1}{81}b_3c_3 + \frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[\frac{1}{36}a_1c_3 + \frac{155}{1944}b_3c_3 - \frac{343}{7776}c_3^2\right]$$

$$+\frac{\Delta^2}{k^{0\,2}}\left[\frac{1}{18}a_1^2-\frac{1}{81}a_1b_3+\frac{1}{108}a_1c_3-\frac{371}{23328}c_3^2\right], \tag{B.28}$$

$$c_{28}^{(\mathrm{s})}=\frac{1}{81}b_3^2-\frac{1}{81}b_3c_3+\frac{1}{162}c_3^2+\frac{\Delta}{k^0}\left[-\frac{49}{648}b_3c_3+\frac{605}{23328}c_3^2\right]+\frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{18}a_1^2-\frac{1}{81}a_1b_3+\frac{1}{36}a_1c_3+\frac{127}{7776}c_3^2\right], \tag{B.29}$$

$$\begin{aligned}c_{29}^{(\mathrm{s})} &= \frac{1}{9}a_1b_3-\frac{1}{18}a_1c_3+\frac{1}{27}b_3^2-\frac{1}{27}b_3c_3+\frac{1}{81}c_3^2+\frac{\Delta}{k^0}\left[\frac{1}{216}a_1c_3+\frac{613}{11664}b_3c_3-\frac{1481}{69984}c_3^2\right]\\ &+\frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{9}a_1^2-\frac{1}{27}a_1b_3+\frac{2}{81}a_1c_3-\frac{149}{17496}c_3^2\right],\end{aligned} \tag{B.30}$$

$$c_{30}^{(\mathrm{s})}=-\frac{1}{9}a_1b_3+\frac{1}{18}a_1c_3-\frac{1}{81}b_3^2+\frac{1}{81}b_3c_3+\frac{\Delta}{k^0}\left[\frac{1}{216}a_1c_3+\frac{613}{11664}b_3c_3-\frac{1733}{69984}c_3^2\right]+\frac{\Delta^2}{k^{0\,2}}\left[\frac{1}{81}a_1b_3-\frac{23}{4374}c_3^2\right], \tag{B.31}$$

$$\begin{aligned}c_{31}^{(\mathrm{s})} &= \frac{1}{18}a_1b_3-\frac{1}{36}a_1c_3+\frac{2}{81}b_3^2-\frac{2}{81}b_3c_3+\frac{1}{108}c_3^2+\frac{\Delta}{k^0}\left[\frac{1}{72}a_1b_3+\frac{5}{216}a_1c_3+\frac{1921}{23328}b_3c_3-\frac{335}{8748}c_3^2\right]\\ &+\frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{18}a_1^2-\frac{2}{81}a_1b_3+\frac{11}{648}a_1c_3-\frac{2051}{139968}c_3^2\right],\end{aligned} \tag{B.32}$$

$$\begin{aligned}c_{32}^{(\mathrm{s})} &= -\frac{1}{18}a_1b_3+\frac{1}{36}a_1c_3-\frac{1}{81}b_3^2+\frac{1}{81}b_3c_3-\frac{1}{324}c_3^2+\frac{\Delta}{k^0}\left[-\frac{1}{72}a_1b_3-\frac{1}{54}a_1c_3-\frac{695}{23328}b_3c_3+\frac{1073}{69984}c_3^2\right]\\ &+\frac{\Delta^2}{k^{0\,2}}\left[\frac{1}{81}a_1b_3-\frac{1}{216}a_1c_3+\frac{1087}{139968}c_3^2\right],\end{aligned} \tag{B.33}$$

$$c_{33}^{(\mathrm{s})}=\frac{\Delta}{k^0}\left[\frac{13}{2916}b_3c_3-\frac{37}{34992}c_3^2\right]+\frac{\Delta^2}{k^{0\,2}}\left[-\frac{23}{17496}c_3^2\right], \tag{B.34}$$

$$c_{34}^{(\mathrm{s})}=\frac{\Delta}{k^0}\left[-\frac{13}{2916}b_3c_3+\frac{37}{34992}c_3^2\right]+\frac{\Delta^2}{k^{0\,2}}\left[\frac{23}{17496}c_3^2\right], \tag{B.35}$$

$$c^{(\mathrm{s})}_{35} = \frac{\Delta}{k^0}\left[\frac{1}{36}a_1 b_2\right], \tag{B.36}$$

$$c^{(\mathrm{s})}_{36} = \frac{\Delta}{k^0}\left[\frac{1}{36}a_1 b_2\right], \tag{B.37}$$

$$c^{(\mathrm{s})}_{37} = \frac{\Delta}{k^0}\left[-\frac{1}{54}a_1 b_3 - \frac{1}{54}a_1 c_3 - \frac{61}{729}b_3 c_3 + \frac{547}{8748}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[\frac{1}{162}a_1 c_3 + \frac{257}{17496}c_3^2\right], \tag{B.38}$$

$$c^{(\mathrm{s})}_{38} = \frac{1}{27}b_2 b_3 - \frac{1}{54}b_2 c_3 + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{54}a_1 b_2\right], \tag{B.39}$$

$$c^{(\mathrm{s})}_{39} = \frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{54}a_1 b_2\right], \tag{B.40}$$

$$c^{(\mathrm{s})}_{40} = \frac{\Delta^2}{k^{0\,2}}\left[\frac{1}{54}a_1 b_2\right], \tag{B.41}$$

$$c^{(\mathrm{s})}_{41} = \frac{\Delta^2}{k^{0\,2}}\left[\frac{1}{54}a_1 b_2\right], \tag{B.42}$$

$$c^{(\mathrm{s})}_{42} = \frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{54}a_1 b_2\right], \tag{B.43}$$

$$c^{(\mathrm{s})}_{43} = \frac{1}{54}b_2 c_3 + \frac{1}{81}b_3 c_3 - \frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[-\frac{31}{1944}b_3 c_3 + \frac{29}{3888}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{2}{81}a_1 c_3 - \frac{1}{243}b_3 c_3 + \frac{11}{5832}c_3^2\right], \tag{B.44}$$

$$\begin{aligned} c^{(\mathrm{s})}_{44} \;=\; & \frac{1}{54}b_2 c_3 + \frac{1}{81}b_3 c_3 - \frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[\frac{31}{1944}b_3 c_3 - \frac{43}{3888}c_3^2\right] \\ & + \frac{\Delta^2}{k^{0\,2}}\left[\frac{1}{27}a_1 b_2 + \frac{2}{81}a_1 b_3 - \frac{1}{81}a_1 c_3 - \frac{1}{729}b_3 c_3 - \frac{1}{729}c_3^2\right], \end{aligned} \tag{B.45}$$

$$c_{45}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[\frac{1}{27}a_1b_3 + \frac{4}{27}a_1c_3 + \frac{491}{2916}b_3c_3 - \frac{191}{2187}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{81}a_1c_3 - \frac{493}{17496}c_3^2\right], \quad \text{(B.46)}$$

$$c_{46}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[-\frac{1}{27}a_1b_3 - \frac{2}{27}a_1c_3 - \frac{491}{2916}b_3c_3 + \frac{209}{2187}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[\frac{1}{81}a_1c_3 + \frac{493}{17496}c_3^2\right], \quad \text{(B.47)}$$

$$c_{47}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[-\frac{1}{54}a_1c_3 - \frac{1}{486}c_3^2\right], \quad \text{(B.48)}$$

$$c_{48}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[-\frac{1}{81}a_1c_3 - \frac{1}{729}c_3^2\right], \quad \text{(B.49)}$$

$$c_{49}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[\frac{1}{1944}b_3c_3 + \frac{1}{2916}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[\frac{7}{11664}c_3^2\right], \quad \text{(B.50)}$$

$$c_{50}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[-\frac{1}{27}a_1b_3 - \frac{1}{24}a_1c_3 - \frac{491}{11664}b_3c_3 + \frac{191}{8748}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[\frac{1}{324}a_1c_3 + \frac{493}{69984}c_3^2\right], \quad \text{(B.51)}$$

$$c_{51}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[\frac{1}{54}a_1b_3 + \frac{1}{72}a_1c_3 + \frac{491}{11664}b_3c_3 - \frac{209}{8748}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{324}a_1c_3 - \frac{493}{69984}c_3^2\right], \quad \text{(B.52)}$$

$$c_{52}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[\frac{1}{216}a_1c_3 + \frac{485}{11664}b_3c_3 - \frac{115}{4374}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{324}a_1c_3 - \frac{535}{69984}c_3^2\right], \quad \text{(B.53)}$$

$$c_{53}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[-\frac{1}{54}a_1b_3 + \frac{1}{216}a_1c_3 - \frac{485}{11664}b_3c_3 + \frac{62}{2187}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[\frac{1}{324}a_1c_3 + \frac{535}{69984}c_3^2\right], \quad \text{(B.54)}$$

$$c_{54}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[-\frac{1}{9}a_1c_3 - \frac{4}{243}c_3^2\right], \quad \text{(B.55)}$$

$$c_{55}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[\frac{1}{27}a_1 b_3 + \frac{1}{54}a_1 c_3 + \frac{1}{243}c_3^2\right], \tag{B.56}$$

$$c_{56}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[\frac{1}{27}a_1 b_3 + \frac{1}{27}a_1 c_3 + \frac{122}{729}b_3 c_3 - \frac{439}{4374}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[-\frac{1}{81}a_1 c_3 - \frac{257}{8748}c_3^2\right], \tag{B.57}$$

$$c_{57}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[-\frac{2}{27}a_1 c_3 - \frac{2}{243}c_3^2\right], \tag{B.58}$$

$$c_{58}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[\frac{1}{27}a_1 c_3 + \frac{1}{243}c_3^2\right], \tag{B.59}$$

$$c_{59}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[\frac{2}{81}a_1 c_3 + \frac{2}{729}c_3^2\right], \tag{B.60}$$

$$c_{60}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[\frac{1}{54}a_1 c_3\right], \tag{B.61}$$

$$c_{61}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[-\frac{1}{54}a_1 c_3\right], \tag{B.62}$$

$$c_{62}^{(\mathrm{s})} = \frac{\Delta^2}{k^{0 2}}\left[\frac{2}{729}b_3 c_3 - \frac{4}{729}c_3^2\right], \tag{B.63}$$

$$c_{63}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[-\frac{7}{972}b_3 c_3 + \frac{17}{1458}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[\frac{1}{729}c_3^2\right], \tag{B.64}$$

$$c_{64}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[\frac{1}{54}b_3 c_3 - \frac{11}{729}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[\frac{1}{729}b_3 c_3 - \frac{2}{729}c_3^2\right], \tag{B.65}$$

$$c_{65}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[-\frac{1}{162}b_3 c_3 - \frac{47}{4374}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[\frac{19}{8748}c_3^2\right], \tag{B.66}$$

$$c_{66}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[-\frac{17}{1458}b_3 c_3 + \frac{131}{8748}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[\frac{1}{324}c_3^2\right], \tag{B.67}$$

$$c_{67}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[\frac{17}{2916}b_3 c_3 - \frac{131}{17496}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[-\frac{1}{648}c_3^2\right], \tag{B.68}$$

$$c^{(\mathrm{s})}_{68} = \frac{\Delta}{k^0}\left[\frac{1}{324}b_3c_3 + \frac{47}{8748}c_3^2\right] + \frac{\Delta^2}{{k^0}^2}\left[-\frac{19}{17496}c_3^2\right], \tag{B.69}$$

$$c^{(\mathrm{s})}_{69} = \frac{1}{162}b_3^2 - \frac{1}{162}b_3c_3 + \frac{1}{648}c_3^2 + \frac{\Delta}{k^0}\left[\frac{7}{7776}c_3^2\right] + \frac{\Delta^2}{{k^0}^2}\left[-\frac{1}{162}a_1b_3 + \frac{1}{324}a_1c_3 - \frac{1}{486}b_3c_3 + \frac{37}{23328}c_3^2\right], \tag{B.70}$$

$$c^{(\mathrm{s})}_{70} = \frac{1}{324}c_3^2 + \frac{\Delta^2}{{k^0}^2}\left[\frac{1}{81}a_1c_3 + \frac{2}{729}b_3c_3 - \frac{1}{729}c_3^2\right], \tag{B.71}$$

$$c^{(\mathrm{s})}_{71} = \frac{\Delta^2}{{k^0}^2}\left[\frac{4}{2187}b_3c_3 - \frac{8}{2187}c_3^2\right], \tag{B.72}$$

$$c^{(\mathrm{s})}_{72} = \frac{\Delta}{k^0}\left[\frac{17}{2916}c_3^2\right] + \frac{\Delta^2}{{k^0}^2}\left[-\frac{2}{729}b_3c_3 + \frac{4}{729}c_3^2\right], \tag{B.73}$$

$$c^{(\mathrm{s})}_{73} = -\frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[-\frac{7}{972}b_3c_3 + \frac{65}{5832}c_3^2\right] + \frac{\Delta^2}{{k^0}^2}\left[-\frac{1}{81}a_1c_3 - \frac{5}{729}c_3^2\right], \tag{B.74}$$

$$c^{(\mathrm{s})}_{74} = -\frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[\frac{7}{972}b_3c_3 - \frac{65}{5832}c_3^2\right] + \frac{\Delta^2}{{k^0}^2}\left[-\frac{1}{27}a_1c_3 - \frac{4}{729}b_3c_3 + \frac{5}{729}c_3^2\right], \tag{B.75}$$

$$c^{(\mathrm{s})}_{75} = -\frac{4}{81}b_3^2 + \frac{4}{81}b_3c_3 - \frac{1}{54}c_3^2 + \frac{\Delta}{k^0}\left[\frac{7}{972}b_3c_3 - \frac{13}{1458}c_3^2\right] + \frac{\Delta^2}{{k^0}^2}\left[\frac{4}{81}a_1b_3 - \frac{5}{81}a_1c_3 + \frac{2}{729}b_3c_3 - \frac{5}{5832}c_3^2\right], \tag{B.76}$$

$$c^{(\mathrm{s})}_{76} = \frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[-\frac{7}{972}b_3c_3 + \frac{31}{5832}c_3^2\right] + \frac{\Delta^2}{{k^0}^2}\left[\frac{1}{27}a_1c_3 + \frac{2}{729}b_3c_3 - \frac{1}{729}c_3^2\right], \tag{B.77}$$

$$c^{(\mathrm{s})}_{77} = \frac{\Delta}{k^0}\left[-\frac{2}{81}b_3c_3 + \frac{11}{729}c_3^2\right] + \frac{\Delta^2}{{k^0}^2}\left[\frac{1}{81}a_1b_3 + \frac{1}{162}a_1c_3 - \frac{1}{729}b_3c_3 + \frac{2}{729}c_3^2\right], \tag{B.78}$$

$$c^{(\mathrm{s})}_{78} = \frac{\Delta}{k^0}\left[\frac{2}{81}b_3c_3 - \frac{41}{1458}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{81}a_1b_3 - \frac{1}{162}a_1c_3 - \frac{1}{729}b_3c_3 + \frac{2}{729}c_3^2\right], \tag{B.79}$$

$$c^{(\mathrm{s})}_{79} = \frac{1}{81}b_3c_3 - \frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[-\frac{31}{1944}b_3c_3 + \frac{29}{3888}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{2}{81}a_1c_3 - \frac{1}{243}b_3c_3 + \frac{11}{5832}c_3^2\right], \tag{B.80}$$

$$c^{(\mathrm{s})}_{80} = -\frac{1}{81}b_3c_3 + \frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[-\frac{31}{1944}b_3c_3 + \frac{43}{3888}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{2}{81}a_1b_3 + \frac{1}{81}a_1c_3 + \frac{1}{729}b_3c_3 + \frac{1}{729}c_3^2\right], \tag{B.81}$$

$$c^{(\mathrm{s})}_{81} = \frac{1}{162}b_3c_3 - \frac{1}{324}c_3^2 + \frac{\Delta}{k^0}\left[-\frac{67}{3888}b_3c_3 + \frac{263}{23328}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{81}a_1c_3 - \frac{2}{729}b_3c_3 + \frac{1}{432}c_3^2\right], \tag{B.82}$$

$$c^{(\mathrm{s})}_{82} = -\frac{1}{162}b_3c_3 + \frac{1}{324}c_3^2 + \frac{\Delta}{k^0}\left[\frac{5}{3888}b_3c_3 - \frac{47}{23328}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{81}a_1b_3 + \frac{1}{162}a_1c_3 + \frac{1}{729}b_3c_3 - \frac{1}{1458}c_3^2\right], \tag{B.83}$$

$$c^{(\mathrm{s})}_{83} = \frac{\Delta}{k^0}\left[\frac{7}{1944}c_3^2\right], \tag{B.84}$$

$$c^{(\mathrm{s})}_{84} = \frac{\Delta}{k^0}\left[-\frac{17}{5832}b_3c_3 + \frac{131}{34992}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[\frac{1}{1296}c_3^2\right], \tag{B.85}$$

$$c^{(\mathrm{s})}_{85} = \frac{\Delta}{k^0}\left[-\frac{1}{648}b_3c_3 - \frac{47}{17496}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[\frac{19}{34992}c_3^2\right], \tag{B.86}$$

$$c^{(\mathrm{s})}_{86} = -\frac{1}{81}b_3^2 + \frac{1}{81}b_3c_3 - \frac{1}{324}c_3^2 + \frac{\Delta}{k^0}\left[-\frac{7}{3888}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[\frac{1}{81}a_1b_3 - \frac{1}{162}a_1c_3 - \frac{13}{11664}c_3^2\right], \tag{B.87}$$

$$c_{87}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[-\frac{1}{1296}b_3c_3 + \frac{1}{864}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[-\frac{1}{162}a_1c_3 + \frac{13}{11664}c_3^2\right], \tag{B.88}$$

$$c_{88}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[\frac{1}{1296}b_3c_3 + \frac{5}{7776}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[-\frac{1}{729}c_3^2\right], \tag{B.89}$$

$$c_{89}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[\frac{1}{1296}b_3c_3 - \frac{37}{7776}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[\frac{1}{162}a_1c_3 - \frac{13}{11664}c_3^2\right], \tag{B.90}$$

$$c_{90}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[-\frac{1}{1296}b_3c_3 + \frac{23}{7776}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[\frac{1}{729}c_3^2\right], \tag{B.91}$$

$$c_{91}^{(\mathrm{s})} = \frac{4}{81}b_3^2 - \frac{4}{81}b_3c_3 + \frac{2}{81}c_3^2 + \frac{\Delta}{k^0}\left[\frac{7}{1944}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[-\frac{4}{81}a_1b_3 + \frac{2}{27}a_1c_3 + \frac{13}{5832}c_3^2\right], \tag{B.92}$$

$$c_{92}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[\frac{1}{324}b_2c_3 - \frac{1}{324}b_3c_3 - \frac{47}{8748}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[\frac{19}{17496}c_3^2\right], \tag{B.93}$$

$$c_{93}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[\frac{1}{324}b_2c_3 + \frac{17}{2916}b_3c_3 - \frac{131}{17496}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[-\frac{1}{648}c_3^2\right], \tag{B.94}$$

$$c_{94}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[-\frac{7}{1944}b_3c_3 + \frac{17}{2916}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[\frac{1}{1458}c_3^2\right], \tag{B.95}$$

$$c_{95}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[\frac{7}{1944}b_3c_3 - \frac{17}{2916}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[-\frac{1}{1458}c_3^2\right], \tag{B.96}$$

$$c_{96}^{(\mathrm{s})} = \frac{\Delta}{k^0}\left[\frac{1}{162}b_3c_3 - \frac{11}{2916}c_3^2\right], \tag{B.97}$$

$$c_{97}^{(\mathrm{s})} = \frac{\Delta^2}{k^{0 2}}\left[-\frac{1}{486}b_2c_3\right], \tag{B.98}$$

$$c_{98}^{(\mathrm{s})} = \frac{\Delta^2}{k^{0 2}}\left[-\frac{1}{486}b_2c_3\right], \tag{B.99}$$

$$c_{99}^{(\mathrm{s})} = \frac{\Delta^2}{k^{0 2}}\left[\frac{1}{486}b_2c_3\right], \tag{B.100}$$

$$c_{100}^{(s)} = \frac{\Delta^2}{k^{0\,2}} \left[\frac{1}{486} b_2 c_3 \right], \tag{B.101}$$

$$c_{101}^{(s)} = \frac{\Delta^2}{k^{0\,2}} \left[-\frac{1}{486} b_2 c_3 \right], \tag{B.102}$$

$$c_{102}^{(s)} = \frac{\Delta^2}{k^{0\,2}} \left[-\frac{1}{729} c_3^2 \right], \tag{B.103}$$

$$c_{103}^{(s)} = \frac{\Delta^2}{k^{0\,2}} \left[\frac{1}{243} b_2 c_3 + \frac{2}{729} b_3 c_3 - \frac{1}{729} c_3^2 \right], \tag{B.104}$$

$$c_{104}^{(s)} = \frac{\Delta}{k^0} \left[-\frac{1}{81} b_3 c_3 + \frac{13}{729} c_3^2 \right], \tag{B.105}$$

$$c_{105}^{(s)} = \frac{\Delta}{k^0} \left[\frac{1}{81} b_3 c_3 - \frac{10}{729} c_3^2 \right], \tag{B.106}$$

$$c_{106}^{(s)} = \frac{\Delta}{k^0} \left[-\frac{1}{972} c_3^2 \right], \tag{B.107}$$

$$c_{107}^{(s)} = \frac{\Delta}{k^0} \left[-\frac{1}{1458} c_3^2 \right], \tag{B.108}$$

$$c_{108}^{(s)} = \frac{\Delta}{k^0} \left[-\frac{1}{486} b_3 c_3 + \frac{7}{2916} c_3^2 \right], \tag{B.109}$$

$$c_{109}^{(s)} = \frac{\Delta}{k^0} \left[-\frac{1}{243} b_3 c_3 + \frac{1}{729} c_3^2 \right], \tag{B.110}$$

$$c_{110}^{(s)} = \frac{\Delta}{k^0} \left[\frac{1}{486} b_3 c_3 - \frac{11}{2916} c_3^2 \right], \tag{B.111}$$

$$c_{111}^{(s)} = \frac{\Delta}{k^0} \left[\frac{4}{729} c_3^2 \right], \tag{B.112}$$

$$c_{112}^{(s)} = \frac{\Delta}{k^0} \left[\frac{1}{243} b_3 c_3 - \frac{5}{729} c_3^2 \right], \tag{B.113}$$

$$c_{113}^{(s)} = \frac{\Delta}{k^0}\left[-\frac{1}{81}b_3c_3 + \frac{11}{1458}c_3^2\right], \tag{B.114}$$

$$c_{114}^{(s)} = \frac{\Delta}{k^0}\left[-\frac{1}{243}c_3^2\right], \tag{B.115}$$

$$c_{115}^{(s)} = \frac{\Delta}{k^0}\left[\frac{1}{486}c_3^2\right], \tag{B.116}$$

$$c_{116}^{(s)} = \frac{\Delta}{k^0}\left[\frac{1}{729}c_3^2\right], \tag{B.117}$$

$$c_{117}^{(s)} = \frac{\Delta}{k^0}\left[\frac{8}{729}c_3^2\right], \tag{B.118}$$

$$c_{118}^{(s)} = \frac{\Delta}{k^0}\left[\frac{13}{1458}c_3^2\right], \tag{B.119}$$

$$c_{119}^{(s)} = \frac{\Delta}{k^0}\left[-\frac{1}{486}c_3^2\right], \tag{B.120}$$

$$c_{120}^{(s)} = 0, \tag{B.121}$$

$$c_{121}^{(s)} = 0, \tag{B.122}$$

$$c_{122}^{(s)} = \frac{\Delta^2}{k^{0^2}}\left[-\frac{1}{1458}b_3c_3 + \frac{1}{2916}c_3^2\right], \tag{B.123}$$

$$c_{123}^{(s)} = \frac{\Delta^2}{k^{0^2}}\left[\frac{1}{1458}c_3^2\right], \tag{B.124}$$

$$c_{124}^{(s)} = 0, \tag{B.125}$$

$$c_{125}^{(s)} = \frac{\Delta^2}{k^{0^2}}\left[-\frac{1}{729}c_3^2\right], \tag{B.126}$$

$$c_{126}^{(\mathrm{s})} = \frac{\Delta^2}{k^{0^2}} \left[-\frac{2}{729} c_3^2 \right], \tag{B.127}$$

$$c_{127}^{(\mathrm{s})} = \frac{\Delta^2}{k^{0^2}} \left[\frac{4}{729} b_3 c_3 - \frac{7}{729} c_3^2 \right], \tag{B.128}$$

$$c_{128}^{(\mathrm{s})} = \frac{\Delta^2}{k^{0^2}} \left[\frac{2}{729} c_3^2 \right], \tag{B.129}$$

$$c_{129}^{(\mathrm{s})} = \frac{\Delta^2}{k^{0^2}} \left[\frac{1}{729} b_3 c_3 \right], \tag{B.130}$$

$$c_{130}^{(\mathrm{s})} = \frac{\Delta^2}{k^{0^2}} \left[-\frac{1}{729} b_3 c_3 \right], \tag{B.131}$$

$$c_{131}^{(\mathrm{s})} = \frac{\Delta^2}{k^{0^2}} \left[-\frac{1}{729} c_3^2 \right], \tag{B.132}$$

$$c_{132}^{(\mathrm{s})} = \frac{\Delta^2}{k^{0^2}} \left[-\frac{2}{729} b_3 c_3 + \frac{1}{729} c_3^2 \right], \tag{B.133}$$

$$c_{133}^{(\mathrm{s})} = \frac{\Delta^2}{k^{0^2}} \left[-\frac{1}{1458} c_3^2 \right], \tag{B.134}$$

$$c_{134}^{(\mathrm{s})} = \frac{\Delta^2}{k^{0^2}} \left[-\frac{1}{729} b_3 c_3 + \frac{1}{1458} c_3^2 \right], \tag{B.135}$$

$$c_{135}^{(\mathrm{s})} = 0, \tag{B.136}$$

$$c_{136}^{(\mathrm{s})} = \frac{\Delta^2}{k^{0^2}} \left[\frac{1}{729} b_3 c_3 - \frac{1}{1458} c_3^2 \right], \tag{B.137}$$

$$c_{137}^{(\mathrm{s})} = \frac{\Delta^2}{k^{0^2}} \left[-\frac{1}{1458} c_3^2 \right], \tag{B.138}$$

$$c_{138}^{(\mathrm{s})} = \frac{\Delta^2}{k^{0^2}} \left[\frac{1}{1458} c_3^2 \right], \tag{B.139}$$

$$c_{139}^{(s)} = \frac{\Delta^2}{k^{0\,2}} \left[-\frac{4}{729} b_3 c_3 + \frac{8}{729} c_3^2 \right], \tag{B.140}$$

$$c_1^{(a)} = 0, \tag{B.141}$$

$$c_2^{(a)} = \frac{1}{6} a_1^2, \tag{B.142}$$

$$c_3^{(a)} = \frac{1}{2} a_1^2, \tag{B.143}$$

$$c_4^{(a)} = 0, \tag{B.144}$$

$$c_5^{(a)} = 0, \tag{B.145}$$

$$c_6^{(a)} = \frac{\Delta}{k^0} \left[-\frac{1}{6} a_1^2 \right], \tag{B.146}$$

$$c_7^{(a)} = \frac{\Delta}{k^0} \left[\frac{1}{6} a_1^2 \right], \tag{B.147}$$

$$c_8^{(a)} = 0, \tag{B.148}$$

$$c_9^{(a)} = \frac{1}{6} a_1 b_2, \tag{B.149}$$

$$c_{10}^{(a)} = \frac{1}{6} a_1 b_2, \tag{B.150}$$

$$c_{11}^{(a)} = 0, \tag{B.151}$$

$$c_{12}^{(a)} = \frac{\Delta}{k^0} \left[\frac{1}{72} a_1 c_3 + \frac{107}{1944} b_3 c_3 - \frac{73}{1944} c_3^2 \right], \tag{B.152}$$

$$c_{13}^{(a)} = \frac{\Delta}{k^0}\left[-\frac{1}{72}a_1c_3 - \frac{19}{324}b_3c_3 + \frac{49}{1296}c_3^2\right], \tag{B.153}$$

$$c_{14}^{(a)} = 0, \tag{B.154}$$

$$c_{15}^{(a)} = \frac{\Delta}{k^0}\left[-\frac{1}{6}a_1^2 - \frac{1}{9}a_1c_3 + \frac{1}{144}b_3c_3 - \frac{5}{2592}c_3^2\right], \tag{B.155}$$

$$c_{16}^{(a)} = \frac{1}{18}a_1c_3 + \frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[\frac{7}{3888}b_3c_3 - \frac{1}{7776}c_3^2\right] + \frac{\Delta^2}{k^{0^2}}\left[\frac{1}{18}a_1^2 + \frac{2}{81}a_1c_3 + \frac{2}{729}c_3^2\right], \tag{B.156}$$

$$c_{17}^{(a)} = \frac{1}{9}a_1c_3 + \frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[-\frac{1}{36}a_1b_3 - \frac{1}{72}a_1c_3 - \frac{935}{7776}b_3c_3 + \frac{13}{192}c_3^2\right] + \frac{\Delta^2}{k^{0^2}}\left[\frac{1}{9}a_1^2 + \frac{7}{324}a_1c_3 + \frac{7}{5832}c_3^2\right], \tag{B.157}$$

$$c_{18}^{(a)} = \frac{1}{36}b_2^2, \tag{B.158}$$

$$c_{19}^{(a)} = \frac{\Delta}{k^0}\left[-\frac{7}{3888}b_3c_3 - \frac{79}{7776}c_3^2\right] + \frac{\Delta^2}{k^{0^2}}\left[-\frac{1}{162}a_1c_3 + \frac{5}{1944}c_3^2\right], \tag{B.159}$$

$$c_{20}^{(a)} = \frac{\Delta}{k^0}\left[-\frac{7}{2592}b_3c_3 + \frac{1}{5184}c_3^2\right], \tag{B.160}$$

$$c_{21}^{(a)} = \frac{\Delta}{k^0}\left[\frac{7}{3888}b_3c_3 - \frac{1}{7776}c_3^2\right], \tag{B.161}$$

$$c_{22}^{(a)} = \frac{1}{27}a_1c_3 + \frac{1}{243}c_3^2 + \frac{\Delta}{k^0}\left[\frac{7}{5832}b_3c_3 - \frac{1}{11664}c_3^2\right] + \frac{\Delta^2}{k^{0^2}}\left[\frac{1}{27}a_1^2 + \frac{4}{243}a_1c_3 + \frac{4}{2187}c_3^2\right], \tag{B.162}$$

$$c_{23}^{(a)} = \frac{1}{18}a_1c_3 + \frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[-\frac{1}{72}a_1c_3 + \frac{205}{7776}b_3c_3 - \frac{115}{5184}c_3^2\right] + \frac{\Delta^2}{k^{0^2}}\left[\frac{11}{324}a_1c_3 + \frac{5}{3888}c_3^2\right], \tag{B.163}$$

$$c_{24}^{(\mathrm{a})} = -\frac{1}{18}a_1c_3 - \frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[\frac{1}{72}a_1c_3 - \frac{233}{7776}b_3c_3 + \frac{349}{15552}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{9}a_1^2 - \frac{1}{108}a_1c_3 - \frac{79}{11664}c_3^2\right], \tag{B.164}$$

$$c_{25}^{(\mathrm{a})} = \frac{1}{18}a_1c_3 + \frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[\frac{1}{72}a_1c_3 - \frac{191}{7776}b_3c_3 + \frac{61}{5184}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[\frac{11}{324}a_1c_3 + \frac{5}{3888}c_3^2\right], \tag{B.165}$$

$$c_{26}^{(\mathrm{a})} = -\frac{1}{18}a_1c_3 - \frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[-\frac{1}{72}a_1c_3 + \frac{73}{2592}b_3c_3 - \frac{187}{15552}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{9}a_1^2 - \frac{1}{108}a_1c_3 - \frac{79}{11664}c_3^2\right], \tag{B.166}$$

$$c_{27}^{(\mathrm{a})} = \frac{1}{9}a_1b_3 - \frac{1}{18}a_1c_3 - \frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[\frac{647}{7776}b_3c_3 - \frac{257}{5184}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{18}a_1^2 - \frac{7}{324}a_1c_3 - \frac{47}{11664}c_3^2\right], \tag{B.167}$$

$$c_{28}^{(\mathrm{a})} = \frac{1}{9}a_1b_3 - \frac{1}{18}a_1c_3 - \frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[-\frac{661}{7776}b_3c_3 + \frac{311}{5184}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{18}a_1^2 - \frac{7}{324}a_1c_3 - \frac{47}{11664}c_3^2\right], \tag{B.168}$$

$$c_{29}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[\frac{1}{72}a_1c_3 + \frac{107}{1944}b_3c_3 - \frac{73}{1944}c_3^2\right], \tag{B.169}$$

$$c_{30}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[\frac{1}{72}a_1c_3 + \frac{19}{324}b_3c_3 - \frac{49}{1296}c_3^2\right], \tag{B.170}$$

$$\begin{aligned} c_{31}^{(\mathrm{a})} &= \frac{1}{18}a_1b_3 - \frac{1}{36}a_1c_3 - \frac{1}{324}c_3^2 + \frac{\Delta}{k^0}\left[\frac{1}{72}a_1b_3 + \frac{1}{72}a_1c_3 + \frac{1363}{15552}b_3c_3 - \frac{1637}{31104}c_3^2\right] \\ &\quad + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{18}a_1^2 - \frac{1}{216}a_1c_3 - \frac{23}{11664}c_3^2\right], \end{aligned} \tag{B.171}$$

$$c_{32}^{(a)} = -\frac{1}{18}a_1 b_3 + \frac{1}{36}a_1 c_3 + \frac{1}{324}c_3^2 + \frac{\Delta}{k^0}\left[-\frac{1}{72}a_1 b_3 - \frac{479}{15552}b_3 c_3 + \frac{155}{10368}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[\frac{11}{648}a_1 c_3 - \frac{1}{1296}c_3^2\right], \tag{B.172}$$

$$c_{33}^{(a)} = \frac{\Delta}{k^0}\left[\frac{1}{243}b_3 c_3 - \frac{13}{3888}c_3^2\right], \tag{B.173}$$

$$c_{34}^{(a)} = \frac{\Delta}{k^0}\left[\frac{1}{18}a_1 b_2 - \frac{1}{243}b_3 c_3 + \frac{13}{3888}c_3^2\right], \tag{B.174}$$

$$c_{35}^{(a)} = \frac{\Delta}{k^0}\left[\frac{1}{36}a_1 b_2\right], \tag{B.175}$$

$$c_{36}^{(a)} = \frac{\Delta}{k^0}\left[-\frac{1}{36}a_1 b_2\right], \tag{B.176}$$

$$c_{37}^{(a)} = \frac{\Delta}{k^0}\left[-\frac{1}{54}a_1 b_3 - \frac{5}{108}a_1 c_3 - \frac{157}{1944}b_3 c_3 + \frac{31}{1296}c_3^2\right], \tag{B.177}$$

$$c_{38}^{(a)} = 0, \tag{B.178}$$

$$c_{39}^{(a)} = \frac{1}{27}b_2 b_3 - \frac{1}{54}b_2 c_3 + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{54}a_1 b_2\right], \tag{B.179}$$

$$c_{40}^{(a)} = \frac{1}{27}b_2 b_3 - \frac{1}{54}b_2 c_3 + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{54}a_1 b_2\right], \tag{B.180}$$

$$c_{41}^{(a)} = \frac{1}{54}b_2 c_3 + \frac{\Delta^2}{k^{0\,2}}\left[\frac{1}{54}a_1 b_2\right], \tag{B.181}$$

$$c_{42}^{(a)} = \frac{1}{54}b_2 c_3 + \frac{\Delta^2}{k^{0\,2}}\left[\frac{1}{54}a_1 b_2\right], \tag{B.182}$$

$$c_{43}^{(a)} = \frac{\Delta}{k^0}\left[-\frac{31}{1944}b_3 c_3 + \frac{11}{1296}c_3^2\right], \tag{B.183}$$

$$c_{44}^{(a)} = \frac{\Delta}{k^0}\left[\frac{19}{972}b_3c_3 - \frac{17}{1944}c_3^2\right], \tag{B.184}$$

$$c_{45}^{(a)} = \frac{\Delta}{k^0}\left[\frac{1}{27}a_1b_3 - \frac{5}{54}a_1c_3 + \frac{323}{1944}b_3c_3 - \frac{335}{3888}c_3^2\right], \tag{B.185}$$

$$c_{46}^{(a)} = \frac{\Delta}{k^0}\left[-\frac{1}{27}a_1b_3 + \frac{5}{54}a_1c_3 - \frac{323}{1944}b_3c_3 + \frac{335}{3888}c_3^2\right], \tag{B.186}$$

$$c_{47}^{(a)} = 0, \tag{B.187}$$

$$c_{48}^{(a)} = 0, \tag{B.188}$$

$$c_{49}^{(a)} = \frac{\Delta}{k^0}\left[\frac{1}{432}b_3c_3 + \frac{1}{288}c_3^2\right], \tag{B.189}$$

$$c_{50}^{(a)} = \frac{\Delta}{k^0}\left[-\frac{1}{27}a_1b_3 + \frac{1}{54}a_1c_3 - \frac{323}{7776}b_3c_3 + \frac{335}{15552}c_3^2\right], \tag{B.190}$$

$$c_{51}^{(a)} = \frac{\Delta}{k^0}\left[\frac{1}{54}a_1b_3 - \frac{1}{108}a_1c_3 + \frac{323}{7776}b_3c_3 - \frac{335}{15552}c_3^2\right], \tag{B.191}$$

$$c_{52}^{(a)} = \frac{\Delta}{k^0}\left[\frac{1}{54}a_1c_3 + \frac{305}{7776}b_3c_3 - \frac{293}{15552}c_3^2\right], \tag{B.192}$$

$$c_{53}^{(a)} = \frac{\Delta}{k^0}\left[\frac{1}{54}a_1b_3 - \frac{1}{36}a_1c_3 - \frac{305}{7776}b_3c_3 + \frac{293}{15552}c_3^2\right], \tag{B.193}$$

$$c_{54}^{(a)} = \frac{\Delta}{k^0}\left[\frac{1}{27}a_1c_3\right], \tag{B.194}$$

$$c_{55}^{(a)} = 0, \tag{B.195}$$

$$c_{56}^{(a)} = \frac{\Delta}{k^0}\left[\frac{1}{27}a_1b_3 + \frac{1}{54}a_1c_3 + \frac{157}{972}b_3c_3 - \frac{157}{1944}c_3^2\right], \tag{B.196}$$

$$c^{(a)}_{57} = 0, \tag{B.197}$$

$$c^{(a)}_{58} = 0, \tag{B.198}$$

$$c^{(a)}_{59} = 0, \tag{B.199}$$

$$c^{(a)}_{60} = \frac{\Delta}{k^0}\left[\frac{1}{54}a_1c_3\right], \tag{B.200}$$

$$c^{(a)}_{61} = \frac{\Delta}{k^0}\left[\frac{1}{54}a_1c_3\right], \tag{B.201}$$

$$c^{(a)}_{62} = \frac{1}{324}c_3^2 + \frac{\Delta^2}{k^{0 2}}\left[\frac{1}{81}a_1c_3 + \frac{1}{243}c_3^2\right], \tag{B.202}$$

$$c^{(a)}_{63} = \frac{\Delta}{k^0}\left[-\frac{7}{972}b_3c_3 + \frac{1}{1944}c_3^2\right], \tag{B.203}$$

$$c^{(a)}_{64} = \frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[\frac{1}{54}b_3c_3 - \frac{17}{1944}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[\frac{2}{81}a_1c_3 + \frac{5}{1458}c_3^2\right], \tag{B.204}$$

$$c^{(a)}_{65} = \frac{\Delta}{k^0}\left[-\frac{2}{243}b_3c_3 + \frac{71}{5832}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[-\frac{11}{8748}c_3^2\right], \tag{B.205}$$

$$c^{(a)}_{66} = \frac{\Delta}{k^0}\left[-\frac{2}{243}b_3c_3 + \frac{7}{5832}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[\frac{11}{8748}c_3^2\right], \tag{B.206}$$

$$c^{(a)}_{67} = \frac{\Delta}{k^0}\left[\frac{1}{243}b_3c_3 - \frac{7}{11664}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[-\frac{11}{17496}c_3^2\right], \tag{B.207}$$

$$c^{(a)}_{68} = \frac{\Delta}{k^0}\left[\frac{1}{243}b_3c_3 - \frac{71}{11664}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[\frac{11}{17496}c_3^2\right], \tag{B.208}$$

$$c^{(a)}_{69} = \frac{\Delta}{k^0}\left[-\frac{7}{7776}b_3c_3 + \frac{1}{15552}c_3^2\right], \tag{B.209}$$

$$c_{70}^{(a)} = 0, \tag{B.210}$$

$$c_{71}^{(a)} = \frac{1}{486}c_3^2 + \frac{\Delta^2}{k^{0 2}}\left[\frac{2}{243}a_1c_3 + \frac{2}{729}c_3^2\right], \tag{B.211}$$

$$c_{72}^{(a)} = \frac{\Delta}{k^0}\left[-\frac{1}{486}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[-\frac{1}{729}c_3^2\right], \tag{B.212}$$

$$c_{73}^{(a)} = \frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[-\frac{7}{972}b_3c_3 - \frac{1}{972}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[\frac{1}{81}a_1c_3 + \frac{11}{1458}c_3^2\right], \tag{B.213}$$

$$c_{74}^{(a)} = -\frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[\frac{7}{972}b_3c_3 + \frac{1}{972}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[-\frac{1}{27}a_1c_3 - \frac{1}{486}c_3^2\right], \tag{B.214}$$

$$c_{75}^{(a)} = \frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[\frac{7}{648}b_3c_3 - \frac{5}{3888}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[\frac{1}{81}a_1c_3 + \frac{11}{1458}c_3^2\right], \tag{B.215}$$

$$c_{76}^{(a)} = -\frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[-\frac{7}{972}b_3c_3 + \frac{1}{972}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[-\frac{1}{27}a_1c_3 - \frac{1}{486}c_3^2\right], \tag{B.216}$$

$$c_{77}^{(a)} = \frac{1}{81}b_3c_3 - \frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[-\frac{2}{81}b_3c_3 + \frac{17}{1944}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[\frac{1}{81}a_1b_3 - \frac{1}{54}a_1c_3 - \frac{5}{1458}c_3^2\right], \tag{B.217}$$

$$c_{78}^{(a)} = \frac{1}{81}b_3c_3 - \frac{1}{162}c_3^2 + \frac{\Delta}{k^0}\left[\frac{2}{81}b_3c_3 - \frac{7}{1944}c_3^2\right] + \frac{\Delta^2}{k^{0 2}}\left[\frac{1}{81}a_1b_3 - \frac{1}{54}a_1c_3 - \frac{5}{1458}c_3^2\right], \tag{B.218}$$

$$c_{79}^{(a)} = \frac{\Delta}{k^0}\left[-\frac{31}{1944}b_3c_3 + \frac{11}{1296}c_3^2\right], \tag{B.219}$$

$$c_{80}^{(a)} = \frac{\Delta}{k^0}\left[-\frac{19}{972}b_3c_3 + \frac{17}{1944}c_3^2\right], \tag{B.220}$$

$$c_{81}^{(\mathrm{a})} = \frac{1}{162}b_3c_3 - \frac{1}{324}c_3^2 + \frac{\Delta}{k^0}\left[-\frac{67}{3888}b_3c_3 + \frac{67}{7776}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{81}a_1c_3 - \frac{1}{972}c_3^2\right], \quad \text{(B.221)}$$

$$c_{82}^{(\mathrm{a})} = -\frac{1}{162}b_3c_3 + \frac{1}{324}c_3^2 + \frac{\Delta}{k^0}\left[-\frac{1}{1944}b_3c_3\right] + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{81}a_1b_3 + \frac{1}{162}a_1c_3 + \frac{7}{2916}c_3^2\right], \quad \text{(B.222)}$$

$$c_{83}^{(\mathrm{a})} = \frac{1}{81}b_3^2 - \frac{1}{81}b_3c_3 + \frac{1}{324}c_3^2 + \frac{\Delta}{k^0}\left[-\frac{1}{648}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{81}a_1b_3 + \frac{1}{162}a_1c_3\right], \quad \text{(B.223)}$$

$$c_{84}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[-\frac{1}{486}b_3c_3 + \frac{7}{23328}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[\frac{11}{34992}c_3^2\right], \quad \text{(B.224)}$$

$$c_{85}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[-\frac{1}{486}b_3c_3 + \frac{71}{23328}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{11}{34992}c_3^2\right], \quad \text{(B.225)}$$

$$c_{86}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[\frac{7}{3888}b_3c_3 - \frac{1}{7776}c_3^2\right], \quad \text{(B.226)}$$

$$c_{87}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[-\frac{1}{1296}b_3c_3 - \frac{1}{2592}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[\frac{1}{162}a_1c_3\right], \quad \text{(B.227)}$$

$$c_{88}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[-\frac{1}{972}b_3c_3 + \frac{1}{1944}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{729}c_3^2\right], \quad \text{(B.228)}$$

$$c_{89}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[\frac{1}{1296}b_3c_3 + \frac{5}{2592}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[\frac{1}{162}a_1c_3\right], \quad \text{(B.229)}$$

$$c_{90}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[\frac{1}{972}b_3c_3 - \frac{1}{486}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{1}{729}c_3^2\right], \quad \text{(B.230)}$$

$$c_{91}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[-\frac{7}{1944}b_3c_3 + \frac{1}{3888}c_3^2\right], \quad \text{(B.231)}$$

$$c_{92}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[\frac{1}{324}b_2c_3 - \frac{1}{243}b_3c_3 + \frac{71}{11664}c_3^2\right] + \frac{\Delta^2}{k^{0\,2}}\left[-\frac{11}{17496}c_3^2\right], \quad \text{(B.232)}$$

$$c_{93}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[-\frac{1}{324}b_2c_3 + \frac{1}{243}b_3c_3 - \frac{7}{11664}c_3^2\right] + \frac{\Delta^2}{{k^0}^2}\left[-\frac{11}{17496}c_3^2\right], \tag{B.233}$$

$$c_{94}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[-\frac{7}{1944}b_3c_3 + \frac{1}{3888}c_3^2\right], \tag{B.234}$$

$$c_{95}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[\frac{1}{162}b_2c_3 + \frac{7}{1944}b_3c_3 - \frac{1}{3888}c_3^2\right], \tag{B.235}$$

$$c_{96}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[\frac{1}{162}b_3c_3 - \frac{5}{972}c_3^2\right], \tag{B.236}$$

$$c_{97}^{(\mathrm{a})} = 0, \tag{B.237}$$

$$c_{98}^{(\mathrm{a})} = \frac{\Delta^2}{{k^0}^2}\left[-\frac{1}{486}b_2c_3\right], \tag{B.238}$$

$$c_{99}^{(\mathrm{a})} = \frac{\Delta^2}{{k^0}^2}\left[-\frac{1}{486}b_2c_3\right], \tag{B.239}$$

$$c_{100}^{(\mathrm{a})} = \frac{\Delta^2}{{k^0}^2}\left[\frac{1}{486}b_2c_3\right], \tag{B.240}$$

$$c_{101}^{(\mathrm{a})} = \frac{\Delta^2}{{k^0}^2}\left[\frac{1}{486}b_2c_3\right], \tag{B.241}$$

$$c_{102}^{(\mathrm{a})} = 0, \tag{B.242}$$

$$c_{103}^{(\mathrm{a})} = 0, \tag{B.243}$$

$$c_{104}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[-\frac{1}{81}b_3c_3 - \frac{1}{243}c_3^2\right], \tag{B.244}$$

$$c_{105}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[\frac{1}{81}b_3c_3 + \frac{1}{243}c_3^2\right], \tag{B.245}$$

$$c_{106}^{(\mathrm{a})} = 0, \tag{B.246}$$

$$c_{107}^{(\mathrm{a})} = 0, \tag{B.247}$$

$$c_{108}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[-\frac{1}{486}b_3c_3\right], \tag{B.248}$$

$$c_{109}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[-\frac{1}{243}b_3c_3 + \frac{1}{324}c_3^2\right], \tag{B.249}$$

$$c_{110}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[\frac{1}{162}b_3c_3 - \frac{1}{324}c_3^2\right], \tag{B.250}$$

$$c_{111}^{(\mathrm{a})} = 0, \tag{B.251}$$

$$c_{112}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[\frac{1}{486}c_3^2\right], \tag{B.252}$$

$$c_{113}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[-\frac{1}{81}b_3c_3 + \frac{1}{162}c_3^2\right], \tag{B.253}$$

$$c_{114}^{(\mathrm{a})} = 0, \tag{B.254}$$

$$c_{115}^{(\mathrm{a})} = 0, \tag{B.255}$$

$$c_{116}^{(\mathrm{a})} = 0, \tag{B.256}$$

$$c_{117}^{(\mathrm{a})} = 0, \tag{B.257}$$

$$c_{118}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[-\frac{1}{486}c_3^2\right], \tag{B.258}$$

$$c_{119}^{(\mathrm{a})} = \frac{\Delta}{k^0}\left[\frac{1}{486}c_3^2\right], \tag{B.259}$$

$$c_{120}^{(\mathrm{a})} = \frac{\Delta^2}{k^{0^2}}\left[\frac{1}{1458}c_3^2\right], \tag{B.260}$$

$$c_{121}^{(\mathrm{a})} = \frac{\Delta^2}{k^{0^2}}\left[\frac{1}{729}c_3^2\right], \tag{B.261}$$

$$c_{122}^{(\mathrm{a})} = 0, \tag{B.262}$$

$$c_{123}^{(\mathrm{a})} = 0, \tag{B.263}$$

$$c_{124}^{(\mathrm{a})} = \frac{\Delta^2}{k^{0^2}}\left[\frac{1}{2187}c_3^2\right], \tag{B.264}$$

$$c_{125}^{(\mathrm{a})} = \frac{\Delta^2}{k^{0^2}}\left[\frac{1}{729}c_3^2\right], \tag{B.265}$$

$$c_{126}^{(\mathrm{a})} = \frac{\Delta^2}{k^{0^2}}\left[-\frac{2}{729}c_3^2\right], \tag{B.266}$$

$$c_{127}^{(\mathrm{a})} = \frac{\Delta^2}{k^{0^2}}\left[\frac{1}{729}c_3^2\right], \tag{B.267}$$

$$c_{128}^{(\mathrm{a})} = \frac{\Delta^2}{k^{0^2}}\left[-\frac{2}{729}c_3^2\right], \tag{B.268}$$

$$c_{129}^{(\mathrm{a})} = \frac{\Delta^2}{k^{0^2}}\left[\frac{1}{729}b_3c_3 - \frac{1}{729}c_3^2\right], \tag{B.269}$$

$$c_{130}^{(\mathrm{a})} = \frac{\Delta^2}{k^{0^2}}\left[\frac{1}{729}b_3c_3 - \frac{1}{729}c_3^2\right], \tag{B.270}$$

$$c_{131}^{(\mathrm{a})} = 0, \tag{B.271}$$

$$c_{132}^{(a)} = 0, \tag{B.272}$$

$$c_{133}^{(a)} = \frac{\Delta^2}{k^{02}} \left[-\frac{1}{1458} c_3^2 \right], \tag{B.273}$$

$$c_{134}^{(a)} = \frac{\Delta^2}{k^{02}} \left[-\frac{1}{729} b_3 c_3 + \frac{1}{1458} c_3^2 \right], \tag{B.274}$$

$$c_{135}^{(a)} = \frac{\Delta^2}{k^{02}} \left[-\frac{1}{729} b_3 c_3 + \frac{1}{1458} c_3^2 \right], \tag{B.275}$$

$$c_{136}^{(a)} = 0, \tag{B.276}$$

$$c_{137}^{(a)} = \frac{\Delta^2}{k^{02}} \left[\frac{1}{1458} c_3^2 \right], \tag{B.277}$$

$$c_{138}^{(a)} = \frac{\Delta^2}{k^{02}} \left[\frac{1}{1458} c_3^2 \right], \tag{B.278}$$

$$c_{139}^{(a)} = 0. \tag{B.279}$$

Non-trivial matrix elements of the form:

Table B.1: Non-trivial matrix elements of operators involved in proton-pion scattering processes

Operator	$p+\pi^+ \to p+\pi^+$	$p+\pi^- \to p+\pi^-$	$p+\pi^0 \to p+\pi^0$	$p+\pi^- \to n+\pi^0$
$k^i k'^j \langle S_1^{(ij)(ab)} \rangle$	$-\mathbf{k}\cdot\mathbf{k}'$	$\mathbf{k}\cdot\mathbf{k}'$	0	$-\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle S_2^{(ij)(ab)} \rangle$	$i(\mathbf{k}\times\mathbf{k}')^3$	$i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^i k'^j \langle S_3^{(ij)(ab)} \rangle$	$\frac{1}{6}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{6}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{12}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^i k'^j \langle S_4^{(ij)(ab)} \rangle$	$\mathbf{k}\cdot\mathbf{k}'$	$\mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	0
$k^i k'^j \langle S_5^{(ij)(ab)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	0
$k^i k'^j \langle S_6^{(ij)(ab)} \rangle$	$\frac{19}{12}\mathbf{k}\cdot\mathbf{k}' - \frac{11}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{19}{12}\mathbf{k}\cdot\mathbf{k}' + \frac{11}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{19}{24}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{11}{12}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle S_7^{(ij)(ab)} \rangle$	$\frac{19}{12}\mathbf{k}\cdot\mathbf{k}' + \frac{11}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{19}{12}\mathbf{k}\cdot\mathbf{k}' - \frac{11}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{19}{24}\mathbf{k}\cdot\mathbf{k}'$	$\frac{11}{12}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle S_8^{(ij)(ab)} \rangle$	$\frac{19}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{19}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{19}{8}\mathbf{k}\cdot\mathbf{k}'$	0
$k^i k'^j \langle S_9^{(ij)(ab)} \rangle$	$\frac{5}{6}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{5}{6}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{5}{12}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^i k'^j \langle S_{10}^{(ij)(ab)} \rangle$	$\frac{5}{6}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{5}{6}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{5}{12}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^i k'^j \langle S_{11}^{(ij)(ab)} \rangle$	$\frac{1}{6}\mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{6}\mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	0
$k^i k'^j \langle S_{12}^{(ij)(ab)} \rangle$	$-\frac{5}{6}\mathbf{k}\cdot\mathbf{k}'$	$\frac{5}{6}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{5}{6}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle S_{13}^{(ij)(ab)} \rangle$	$-\frac{5}{6}\mathbf{k}\cdot\mathbf{k}'$	$\frac{5}{6}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{5}{6}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle S_{14}^{(ij)(ab)} \rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	0
$k^i k'^j \langle S_{15}^{(ij)(ab)} \rangle$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle S_{16}^{(ij)(ab)} \rangle$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{4}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^i k'^j \langle S_{18}^{(ij)(ab)} \rangle$	$i(\mathbf{k}\times\mathbf{k}')^3$	$i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^i k'^j \langle S_{19}^{(ij)(ab)} \rangle$	$\frac{19}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{19}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{19}{8}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^i k'^j \langle S_{20}^{(ij)(ab)} \rangle$	$-\mathbf{k}\cdot\mathbf{k}'$	$\mathbf{k}\cdot\mathbf{k}'$	0	$-\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle S_{23}^{(ij)(ab)} \rangle$	$-\frac{4}{3}\mathbf{k}\cdot\mathbf{k}' - \frac{4}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{4}{3}\mathbf{k}\cdot\mathbf{k}' - \frac{4}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{2}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{4}{3}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle S_{24}^{(ij)(ab)} \rangle$	$-\frac{4}{3}\mathbf{k}\cdot\mathbf{k}' + \frac{4}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{4}{3}\mathbf{k}\cdot\mathbf{k}' + \frac{4}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{2}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{4}{3}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle S_{25}^{(ij)(ab)} \rangle$	$\frac{4}{3}\mathbf{k}\cdot\mathbf{k}' - \frac{4}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{4}{3}\mathbf{k}\cdot\mathbf{k}' - \frac{4}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{2}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{4}{3}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle S_{26}^{(ij)(ab)} \rangle$	$\frac{4}{3}\mathbf{k}\cdot\mathbf{k}' + \frac{4}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{4}{3}\mathbf{k}\cdot\mathbf{k}' + \frac{4}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{2}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{4}{3}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle S_{27}^{(ij)(ab)} \rangle$	$\frac{25}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{25}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{25}{24}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^i k'^j \langle S_{28}^{(ij)(ab)} \rangle$	$\frac{25}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{25}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{25}{24}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^i k'^j \langle S_{29}^{(ij)(ab)} \rangle$	$-\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle S_{30}^{(ij)(ab)} \rangle$	$\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle S_{38}^{(ij)(ab)} \rangle$	$-\frac{5}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{5}{2}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{5}{2}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle S_{39}^{(ij)(ab)} \rangle$	$\frac{5}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{5}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{5}{8}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^i k'^j \langle S_{40}^{(ij)(ab)} \rangle$	$\frac{5}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{5}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{5}{8}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^i k'^j \langle S_{47}^{(ij)(ab)} \rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	0
$k^i k'^j \langle S_{49}^{(ij)(ab)} \rangle$	$4i(\mathbf{k}\times\mathbf{k}')^3$	$-4i(\mathbf{k}\times\mathbf{k}')^3$	0	$4i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle S_{54}^{(ij)(ab)} \rangle$	$4\mathbf{k}\cdot\mathbf{k}' - i(\mathbf{k}\times\mathbf{k}')^3$	$4\mathbf{k}\cdot\mathbf{k}' + i(\mathbf{k}\times\mathbf{k}')^3$	$2\mathbf{k}\cdot\mathbf{k}'$	$-i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle S_{56}^{(ij)(ab)} \rangle$	$\frac{25}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{25}{8}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{25}{8}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle S_{60}^{(ij)(ab)} \rangle$	$-8\mathbf{k}\cdot\mathbf{k}' + 2i(\mathbf{k}\times\mathbf{k}')^3$	$-8\mathbf{k}\cdot\mathbf{k}' - 2i(\mathbf{k}\times\mathbf{k}')^3$	$-4\mathbf{k}\cdot\mathbf{k}'$	$2i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle S_{67}^{(ij)(ab)} \rangle$	$\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{5}{8}\mathbf{k}\cdot\mathbf{k}'$	0
$k^i k'^j \langle S_{68}^{(ij)(ab)} \rangle$	$\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{5}{8}\mathbf{k}\cdot\mathbf{k}'$	0
$k^i k'^j \langle S_{83}^{(ij)(ab)} \rangle$	$\frac{25}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{25}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{25}{8}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^i k'^j \langle S_{84}^{(ij)(ab)} \rangle$	$\frac{5}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{5}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$	0
$k^i k'^j \langle S_{85}^{(ij)(ab)} \rangle$	$\frac{5}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{5}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$	0
$k^i k'^j \langle S_{86}^{(ij)(ab)} \rangle$	$4\mathbf{k}\cdot\mathbf{k}'$	$-4\mathbf{k}\cdot\mathbf{k}'$	0	$4\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle S_{91}^{(ij)(ab)} \rangle$	$6\mathbf{k}\cdot\mathbf{k}' - 6i(\mathbf{k}\times\mathbf{k}')^3$	$-6\mathbf{k}\cdot\mathbf{k}' - 6i(\mathbf{k}\times\mathbf{k}')^3$	$-3i(\mathbf{k}\times\mathbf{k}')^3$	$6\mathbf{k}\cdot\mathbf{k}'$

Table B.2: Non-trivial matrix elements of operators involved in neutron-pion scattering processes

Operator	$n+\pi^+ \to n+\pi^+$	$n+\pi^- \to n+\pi^-$	$n+\pi^0 \to n+\pi^0$	$n+\pi^+ \to p+\pi^0$
$k^ik'^j\langle S_1^{(ij)(ab)}\rangle$	$\mathbf{k}\cdot\mathbf{k}'$	$-\mathbf{k}\cdot\mathbf{k}'$	0	$-\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle S_2^{(ij)(ab)}\rangle$	$i(\mathbf{k}\times\mathbf{k}')^3$	$i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^ik'^j\langle S_3^{(ij)(ab)}\rangle$	$\frac{1}{6}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{6}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{12}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^ik'^j\langle S_4^{(ij)(ab)}\rangle$	$\mathbf{k}\cdot\mathbf{k}'$	$\mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	0
$k^ik'^j\langle S_5^{(ij)(ab)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	0
$k^ik'^j\langle S_6^{(ij)(ab)}\rangle$	$\frac{19}{12}\mathbf{k}\cdot\mathbf{k}'+\frac{11}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{19}{12}\mathbf{k}\cdot\mathbf{k}'-\frac{11}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{19}{24}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{11}{12}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle S_7^{(ij)(ab)}\rangle$	$\frac{19}{12}\mathbf{k}\cdot\mathbf{k}'-\frac{11}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{19}{12}\mathbf{k}\cdot\mathbf{k}'+\frac{11}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{19}{24}\mathbf{k}\cdot\mathbf{k}'$	$\frac{11}{12}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle S_8^{(ij)(ab)}\rangle$	$\frac{19}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{19}{4}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{19}{8}\mathbf{k}\cdot\mathbf{k}'$	0
$k^ik'^j\langle S_9^{(ij)(ab)}\rangle$	$\frac{5}{6}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{5}{6}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{5}{12}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^ik'^j\langle S_{10}^{(ij)(ab)}\rangle$	$\frac{5}{6}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{5}{6}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{5}{12}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^ik'^j\langle S_{11}^{(ij)(ab)}\rangle$	$\frac{1}{6}\mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{6}\mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	0
$k^ik'^j\langle S_{12}^{(ij)(ab)}\rangle$	$\frac{5}{6}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{5}{6}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{5}{6}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle S_{13}^{(ij)(ab)}\rangle$	$\frac{5}{6}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{5}{6}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{5}{6}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle S_{14}^{(ij)(ab)}\rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	0
$k^ik'^j\langle S_{15}^{(ij)(ab)}\rangle$	$-\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle S_{16}^{(ij)(ab)}\rangle$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{4}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^ik'^j\langle S_{18}^{(ij)(ab)}\rangle$	$i(\mathbf{k}\times\mathbf{k}')^3$	$i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^ik'^j\langle S_{19}^{(ij)(ab)}\rangle$	$\frac{19}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{19}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{19}{8}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^ik'^j\langle S_{20}^{(ij)(ab)}\rangle$	$\mathbf{k}\cdot\mathbf{k}'$	$-\mathbf{k}\cdot\mathbf{k}'$	0	$-\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle S_{23}^{(ij)(ab)}\rangle$	$\frac{4}{3}\mathbf{k}\cdot\mathbf{k}'-\frac{4}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{4}{3}\mathbf{k}\cdot\mathbf{k}'-\frac{4}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{2}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{4}{3}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle S_{24}^{(ij)(ab)}\rangle$	$\frac{4}{3}\mathbf{k}\cdot\mathbf{k}'+\frac{4}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{4}{3}\mathbf{k}\cdot\mathbf{k}'+\frac{4}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{2}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{4}{3}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle S_{25}^{(ij)(ab)}\rangle$	$-\frac{4}{3}\mathbf{k}\cdot\mathbf{k}'-\frac{4}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{4}{3}\mathbf{k}\cdot\mathbf{k}'-\frac{4}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{2}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{4}{3}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle S_{26}^{(ij)(ab)}\rangle$	$-\frac{4}{3}\mathbf{k}\cdot\mathbf{k}'+\frac{4}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{4}{3}\mathbf{k}\cdot\mathbf{k}'+\frac{4}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{2}{3}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{4}{3}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle S_{27}^{(ij)(ab)}\rangle$	$\frac{25}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{25}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{25}{24}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^ik'^j\langle S_{28}^{(ij)(ab)}\rangle$	$\frac{25}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{25}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{25}{24}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^ik'^j\langle S_{29}^{(ij)(ab)}\rangle$	$\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle S_{30}^{(ij)(ab)}\rangle$	$-\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle S_{38}^{(ij)(ab)}\rangle$	$\frac{5}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{5}{2}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{5}{2}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle S_{39}^{(ij)(ab)}\rangle$	$\frac{5}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{5}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{5}{8}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^ik'^j\langle S_{40}^{(ij)(ab)}\rangle$	$\frac{5}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{5}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{5}{8}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^ik'^j\langle S_{47}^{(ij)(ab)}\rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	0
$k^ik'^j\langle S_{49}^{(ij)(ab)}\rangle$	$-4i(\mathbf{k}\times\mathbf{k}')^3$	$4i(\mathbf{k}\times\mathbf{k}')^3$	0	$4i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle S_{54}^{(ij)(ab)}\rangle$	$4\mathbf{k}\cdot\mathbf{k}'+i(\mathbf{k}\times\mathbf{k}')^3$	$4\mathbf{k}\cdot\mathbf{k}'-i(\mathbf{k}\times\mathbf{k}')^3$	$2\mathbf{k}\cdot\mathbf{k}'$	$-i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle S_{56}^{(ij)(ab)}\rangle$	$-\frac{25}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{25}{8}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{25}{8}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle S_{60}^{(ij)(ab)}\rangle$	$-8\mathbf{k}\cdot\mathbf{k}'-2i(\mathbf{k}\times\mathbf{k}')^3$	$-8\mathbf{k}\cdot\mathbf{k}'+2i(\mathbf{k}\times\mathbf{k}')^3$	$-4\mathbf{k}\cdot\mathbf{k}'$	$2i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle S_{67}^{(ij)(ab)}\rangle$	$\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{5}{8}\mathbf{k}\cdot\mathbf{k}'$	0
$k^ik'^j\langle S_{68}^{(ij)(ab)}\rangle$	$\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{5}{8}\mathbf{k}\cdot\mathbf{k}'$	0
$k^ik'^j\langle S_{83}^{(ij)(ab)}\rangle$	$\frac{25}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{25}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{25}{8}i(\mathbf{k}\times\mathbf{k}')^3$	0
$k^ik'^j\langle S_{84}^{(ij)(ab)}\rangle$	$\frac{5}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{5}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$	0
$k^ik'^j\langle S_{85}^{(ij)(ab)}\rangle$	$\frac{5}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{5}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{5}{4}\mathbf{k}\cdot\mathbf{k}'$	0
$k^ik'^j\langle S_{86}^{(ij)(ab)}\rangle$	$-4\mathbf{k}\cdot\mathbf{k}'$	$4\mathbf{k}\cdot\mathbf{k}'$	0	$4\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle S_{91}^{(ij)(ab)}\rangle$	$-6\mathbf{k}\cdot\mathbf{k}'-6i(\mathbf{k}\times\mathbf{k}')^3$	$6\mathbf{k}\cdot\mathbf{k}'-6i(\mathbf{k}\times\mathbf{k}')^3$	$-3i(\mathbf{k}\times\mathbf{k}')^3$	$6\mathbf{k}\cdot\mathbf{k}'$

Appendix C

Operators Basis for First Order SB

C.1 Operator basis contributing to SB effects

A list of spin-2 operators containing three adjoint indices is provided. These linearly independent operators are used to build the different $1/N_c$ expansions responsible for SB to the scattering amplitude. It is important to remark that there is no a particular criterion to rule out any of those operators and the reduction rules derived in [5] do not apply for all of these operators.

$$R_1^{(ij)(ab8)} = i\delta^{ij} f^{ab8}, \qquad R_2^{(ij)(ab8)} = \delta^{ij} d^{ab8},$$

$$R_3^{(ij)(ab8)} = \epsilon^{ijm} f^{ab8} J^m, \qquad R_4^{(ij)(ab8)} = i\epsilon^{ijm} d^{ab8} J^m,$$

$$R_5^{(ij)(ab8)} = \delta^{ij} \delta^{ab} T^8, \qquad R_6^{(ij)(ab8)} = \delta^{ij} \delta^{a8} T^b,$$

$$R_7^{(ij)(ab8)} = \delta^{ij} \delta^{b8} T^a, \qquad R_8^{(ij)(ab8)} = i\epsilon^{ijm} \delta^{ab} G^{m8},$$

$$R_9^{(ij)(ab8)} = i\epsilon^{ijm} \delta^{a8} G^{mb}, \qquad R_{10}^{(ij)(ab8)} = i\epsilon^{ijm} \delta^{b8} G^{ma},$$

$$R_{11}^{(ij)(ab8)} = i\epsilon^{ijm} f^{abe} f^{8eg} G^{mg}, \qquad R_{12}^{(ij)(ab8)} = i\epsilon^{ijm} f^{a8e} f^{beg} G^{mg},$$

$$R_{13}^{(ij)(ab8)} = i\epsilon^{ijm} d^{abe} d^{8eg} G^{mg}, \qquad R_{14}^{(ij)(ab8)} = \epsilon^{ijm} f^{abe} d^{8eg} G^{mg},$$

$$R_{15}^{(ij)(ab8)} = \epsilon^{ijm} f^{a8e} d^{beg} G^{mg}, \qquad R_{16}^{(ij)(ab8)} = \epsilon^{ijm} d^{abe} f^{8eg} G^{mg},$$

$$R_{17}^{(ij)(ab8)} = i f^{ab8} \{J^i, J^j\}, \qquad R_{18}^{(ij)(ab8)} = d^{ab8} \{J^i, J^j\},$$

$$R_{19}^{(ij)(ab8)} = i\delta^{ij} f^{ab8} J^2, \qquad R_{20}^{(ij)(ab8)} = \delta^{ij} d^{ab8} J^2,$$

$$R_{21}^{(ij)(ab8)} = i\epsilon^{ijm} \delta^{ab} \mathcal{D}_2^{m8}, \qquad R_{22}^{(ij)(ab8)} = i\epsilon^{ijm} \delta^{a8} \mathcal{D}_2^{mb},$$

$$R_{23}^{(ij)(ab8)} = i\epsilon^{ijm}\delta^{b8}\mathcal{D}_2^{ma},$$
$$R_{24}^{(ij)(ab8)} = i\epsilon^{ijm}f^{abe}f^{8eg}\mathcal{D}_2^{mg},$$
$$R_{25}^{(ij)(ab8)} = i\epsilon^{ijm}f^{a8e}f^{beg}\mathcal{D}_2^{mg},$$
$$R_{26}^{(ij)(ab8)} = i\epsilon^{ijm}d^{abe}d^{8eg}\mathcal{D}_2^{mg},$$
$$R_{27}^{(ij)(ab8)} = \epsilon^{ijm}f^{abe}d^{8eg}\mathcal{D}_2^{mg},$$
$$R_{28}^{(ij)(ab8)} = \epsilon^{ijm}f^{a8e}d^{beg}\mathcal{D}_2^{mg},$$
$$R_{29}^{(ij)(ab8)} = \epsilon^{ijm}f^{b8e}d^{aeg}\mathcal{D}_2^{mg},$$
$$R_{30}^{(ij)(ab8)} = \delta^{ab}\{J^i, G^{j8}\},$$
$$R_{31}^{(ij)(ab8)} = \delta^{a8}\{J^i, G^{jb}\},$$
$$R_{32}^{(ij)(ab8)} = \delta^{b8}\{J^i, G^{ja}\},$$
$$R_{33}^{(ij)(ab8)} = \delta^{ab}\{J^j, G^{i8}\},$$
$$R_{34}^{(ij)(ab8)} = \delta^{a8}\{J^j, G^{ib}\},$$
$$R_{35}^{(ij)(ab8)} = \delta^{b8}\{J^j, G^{ia}\},$$
$$R_{36}^{(ij)(ab8)} = f^{abe}f^{8eg}\{J^i, G^{jg}\},$$
$$R_{37}^{(ij)(ab8)} = f^{a8e}f^{beg}\{J^i, G^{jg}\},$$
$$R_{38}^{(ij)(ab8)} = f^{abe}f^{8eg}\{J^j, G^{ig}\},$$
$$R_{39}^{(ij)(ab8)} = f^{a8e}f^{beg}\{J^j, G^{ig}\},$$
$$R_{40}^{(ij)(ab8)} = d^{abe}d^{8eg}\{J^i, G^{jg}\},$$
$$R_{41}^{(ij)(ab8)} = d^{abe}d^{8eg}\{J^j, G^{ig}\},$$
$$R_{42}^{(ij)(ab8)} = if^{abe}d^{8eg}\{J^i, G^{jg}\},$$
$$R_{43}^{(ij)(ab8)} = if^{a8e}d^{beg}\{J^i, G^{jg}\},$$
$$R_{44}^{(ij)(ab8)} = if^{b8e}d^{aeg}\{J^i, G^{jg}\},$$
$$R_{45}^{(ij)(ab8)} = if^{abe}d^{8eg}\{J^j, G^{ig}\},$$
$$R_{46}^{(ij)(ab8)} = if^{8ae}d^{beg}\{J^j, G^{ig}\},$$
$$R_{47}^{(ij)(ab8)} = if^{b8e}d^{aeg}\{J^j, G^{ig}\},$$
$$R_{48}^{(ij)(ab8)} = \delta^{ij}\delta^{ab}d^{8eg}\{T^e, T^g\},$$
$$R_{49}^{(ij)(ab8)} = \delta^{ij}\delta^{a8}d^{beg}\{T^e, T^g\},$$
$$R_{50}^{(ij)(ab8)} = \delta^{ij}\delta^{b8}d^{aeg}\{T^e, T^g\},$$
$$R_{51}^{(ij)(ab8)} = i\delta^{ij}f^{abe}\{T^8, T^e\},$$
$$R_{52}^{(ij)(ab8)} = \delta^{ij}d^{abe}\{T^8, T^e\},$$
$$R_{53}^{(ij)(ab8)} = \delta^{ij}d^{a8e}\{T^b, T^e\},$$
$$R_{54}^{(ij)(ab8)} = \delta^{ij}d^{b8e}\{T^a, T^e\},$$
$$R_{55}^{(ij)(ab8)} = \delta^{ij}[T^a, \{T^b, T^8\}],$$
$$R_{56}^{(ij)(ab8)} = \delta^{ij}[T^8, \{T^a, T^b\}],$$
$$R_{57}^{(ij)(ab8)} = if^{abe}[J^i, \{T^8, G^{je}\}],$$
$$R_{58}^{(ij)(ab8)} = if^{a8e}[J^i, \{T^b, G^{je}\}],$$
$$R_{59}^{(ij)(ab8)} = if^{b8e}[J^i, \{T^a, G^{je}\}],$$
$$R_{60}^{(ij)(ab8)} = if^{abe}[J^i, \{T^e, G^{j8}\}],$$
$$R_{61}^{(ij)(ab8)} = if^{a8e}[J^i, \{T^e, G^{jb}\}],$$
$$R_{62}^{(ij)(ab8)} = if^{b8e}[J^i, \{T^e, G^{ja}\}],$$
$$R_{63}^{(ij)(ab8)} = d^{abe}[J^i, \{T^8, G^{je}\}],$$
$$R_{64}^{(ij)(ab8)} = d^{a8e}[J^i, \{T^b, G^{je}\}],$$
$$R_{65}^{(ij)(ab8)} = d^{b8e}[J^i, \{T^a, G^{je}\}],$$
$$R_{66}^{(ij)(ab8)} = d^{abe}[J^i, \{T^e, G^{j8}\}],$$
$$R_{67}^{(ij)(ab8)} = d^{a8e}[J^i, \{T^e, G^{jb}\}],$$
$$R_{68}^{(ij)(ab8)} = d^{b8e}[J^i, \{T^e, G^{ja}\}],$$
$$R_{69}^{(ij)(ab8)} = i\epsilon^{ijm}[G^{ma}, \{G^{rb}, G^{r8}\}],$$
$$R_{70}^{(ij)(ab8)} = i\epsilon^{ijm}[G^{m8}, \{G^{ra}, G^{rb}\}],$$
$$R_{71}^{(ij)(ab8)} = i\epsilon^{ijm}[G^{mb}, \{G^{r8}, G^{ra}\}],$$
$$R_{72}^{(ij)(ab8)} = \epsilon^{ijm}f^{ab8}\{J^2, J^m\},$$
$$R_{73}^{(ij)(ab8)} = i\epsilon^{ijm}d^{ab8}\{J^2, J^m\},$$
$$R_{74}^{(ij)(ab8)} = \delta^{ij}\delta^{ab}\{J^2, T^8\},$$
$$R_{75}^{(ij)(ab8)} = \delta^{ij}\delta^{a8}\{J^2, T^b\},$$
$$R_{76}^{(ij)(ab8)} = \delta^{ij}\delta^{b8}\{J^2, T^a\},$$
$$R_{77}^{(ij)(ab8)} = \epsilon^{ijm}f^{abe}\{J^m, \{T^e, T^8\}\},$$
$$R_{78}^{(ij)(ab8)} = \epsilon^{ijm}f^{a8e}\{J^m, \{T^e, T^b\}\},$$

$$R_{79}^{(ij)(ab8)} = \epsilon^{ijm} f^{b8e}\{J^m, \{T^e, T^a\}\}, \qquad R_{80}^{(ij)(ab8)} = i\epsilon^{ijm} d^{abe}\{J^m, \{T^e, T^8\}\},$$
$$R_{81}^{(ij)(ab8)} = i\epsilon^{ijm} d^{a8e}\{J^m, \{T^e, T^b\}\}, \qquad R_{82}^{(ij)(ab8)} = i\epsilon^{ijm} d^{b8e}\{J^m, \{T^e, T^a\}\},$$
$$R_{83}^{(ij)(ab8)} = if^{abe}\{J^i, \{T^8, G^{je}\}\}, \qquad R_{84}^{(ij)(ab8)} = if^{a8e}\{J^i, \{T^b, G^{je}\}\},$$
$$R_{85}^{(ij)(ab8)} = if^{b8e}\{J^i, \{T^a, G^{je}\}\}, \qquad R_{86}^{(ij)(ab8)} = if^{abe}\{J^j, \{T^8, G^{ie}\}\},$$
$$R_{87}^{(ij)(ab8)} = if^{a8e}\{J^j, \{T^b, G^{ie}\}\}, \qquad R_{88}^{(ij)(ab8)} = if^{b8e}\{J^j, \{T^a, G^{ie}\}\},$$
$$R_{89}^{(ij)(ab8)} = if^{abe}\{J^i, \{T^e, G^{j8}\}\}, \qquad R_{90}^{(ij)(ab8)} = if^{a8e}\{J^i, \{T^e, G^{jb}\}\},$$
$$R_{91}^{(ij)(ab8)} = if^{b8e}\{J^i, \{T^e, G^{ja}\}\}, \qquad R_{92}^{(ij)(ab8)} = if^{abe}\{J^j, \{T^e, G^{i8}\}\},$$
$$R_{93}^{(ij)(ab8)} = if^{a8e}\{J^j, \{T^e, G^{ib}\}\}, \qquad R_{94}^{(ij)(ab8)} = if^{b8e}\{J^j, \{T^e, G^{ia}\}\},$$
$$R_{95}^{(ij)(ab8)} = d^{abe}\{J^i, \{T^e, G^{j8}\}\}, \qquad R_{96}^{(ij)(ab8)} = d^{a8e}\{J^i, \{T^e, G^{jb}\}\},$$
$$R_{97}^{(ij)(ab8)} = d^{b8e}\{J^i, \{T^e, G^{ja}\}\}, \qquad R_{98}^{(ij)(ab8)} = d^{abe}\{J^j, \{T^e, G^{i8}\}\},$$
$$R_{99}^{(ij)(ab8)} = d^{a8e}\{J^j, \{T^e, G^{ib}\}\}, \qquad R_{100}^{(ij)(ab8)} = d^{b8e}\{J^j, \{T^e, G^{ia}\}\},$$
$$R_{101}^{(ij)(ab8)} = i\epsilon^{ijm}\delta^{ab}\mathcal{D}_3^{m8}, \qquad R_{102}^{(ij)(ab8)} = i\epsilon^{ijm}\delta^{a8}\mathcal{D}_3^{mb},$$
$$R_{103}^{(ij)(ab8)} = i\epsilon^{ijm}\delta^{b8}\mathcal{D}_3^{ma}, \qquad R_{104}^{(ij)(ab8)} = i\epsilon^{ijm}\delta^{ab}\mathcal{O}_3^{m8},$$
$$R_{105}^{(ij)(ab8)} = i\epsilon^{ijm}\delta^{a8}\mathcal{O}_3^{mb}, \qquad R_{106}^{(ij)(ab8)} = i\epsilon^{ijm}\delta^{b8}\mathcal{O}_3^{ma},$$
$$R_{107}^{(ij)(ab8)} = \delta^{ij}\{T^a, \{T^b, T^8\}\}, \qquad R_{108}^{(ij)(ab8)} = \delta^{ij}\{T^8, \{T^a, T^b\}\},$$
$$R_{109}^{(ij)(ab8)} = \delta^{ij}\{T^b, \{T^8, T^a\}\}, \qquad R_{110}^{(ij)(ab8)} = \delta^{ij}\{T^a, \{G^{rb}, G^{r8}\}\},$$
$$R_{111}^{(ij)(ab8)} = \delta^{ij}\{T^8, \{G^{ra}, G^{rb}\}\}, \qquad R_{112}^{(ij)(ab8)} = \delta^{ij}\{T^b, \{G^{r8}, G^{ra}\}\},$$
$$R_{113}^{(ij)(ab8)} = \delta^{ab}\{T^8, \{G^{ie}, G^{je}\}\}, \qquad R_{114}^{(ij)(ab8)} = \delta^{a8}\{T^b, \{G^{ie}, G^{je}\}\},$$
$$R_{115}^{(ij)(ab8)} = \delta^{b8}\{T^a, \{G^{ie}, G^{je}\}\}, \qquad R_{116}^{(ij)(ab8)} = \{T^a, \{G^{ib}, G^{j8}\}\},$$
$$R_{117}^{(ij)(ab8)} = \{T^8, \{G^{ia}, G^{jb}\}\}, \qquad R_{118}^{(ij)(ab8)} = \{T^b, \{G^{i8}, G^{ja}\}\},$$
$$R_{119}^{(ij)(ab8)} = \{T^a, \{G^{jb}, G^{i8}\}\}, \qquad R_{120}^{(ij)(ab8)} = \{T^8, \{G^{ja}, G^{ib}\}\},$$
$$R_{121}^{(ij)(ab8)} = \{T^b, \{G^{j8}, G^{ia}\}\}, \qquad R_{122}^{(ij)(ab8)} = i\epsilon^{ijm}\{G^{ma}, \{T^b, T^8\}\},$$
$$R_{123}^{(ij)(ab8)} = i\epsilon^{ijm}\{G^{m8}, \{T^a, T^b\}\}, \qquad R_{124}^{(ij)(ab8)} = i\epsilon^{ijm}\{G^{mb}, \{T^8, T^a\}\},$$
$$R_{125}^{(ij)(ab8)} = i\epsilon^{ijm} f^{aeg} f^{beh}\{G^{m8}, \{T^g, T^h\}\}, \qquad R_{126}^{(ij)(ab8)} = i\epsilon^{ijm} f^{8eg} f^{aeh}\{G^{mb}, \{T^g, T^h\}\},$$
$$R_{127}^{(ij)(ab8)} = i\epsilon^{ijm} f^{beg} f^{8eh}\{G^{ma}, \{T^g, T^h\}\}, \qquad R_{128}^{(ij)(ab8)} = i\epsilon^{ijm} d^{aeg} d^{beh}\{G^{m8}, \{T^g, T^h\}\},$$
$$R_{129}^{(ij)(ab8)} = i\epsilon^{ijm} d^{8eg} d^{aeh}\{G^{mb}, \{T^g, T^h\}\}, \qquad R_{130}^{(ij)(ab8)} = i\epsilon^{ijm} d^{beg} d^{8eh}\{G^{ma}, \{T^g, T^h\}\},$$
$$R_{131}^{(ij)(ab8)} = \epsilon^{ijm} f^{aeg} d^{beh}\{G^{m8}, \{T^g, T^h\}\}, \qquad R_{132}^{(ij)(ab8)} = \epsilon^{ijm} f^{8eg} d^{aeh}\{G^{mb}, \{T^g, T^h\}\},$$
$$R_{133}^{(ij)(ab8)} = \epsilon^{ijm} f^{beg} d^{8eh}\{G^{ma}, \{T^g, T^h\}\}, \qquad R_{134}^{(ij)(ab8)} = f^{aeg} f^{beh}\{T^8, \{G^{ig}, G^{jh}\}\},$$

$$R_{135}^{(ij)(ab8)} = f^{aeg} f^{beh} \{T^h, \{G^{i8}, G^{jg}\}\}, \qquad R_{136}^{(ij)(ab8)} = f^{aeg} f^{beh} \{T^g, \{G^{ih}, G^{j8}\}\},$$
$$R_{137}^{(ij)(ab8)} = f^{8eg} f^{aeh} \{T^b, \{G^{ig}, G^{jh}\}\}, \qquad R_{138}^{(ij)(ab8)} = f^{8eg} f^{aeh} \{T^h, \{G^{ib}, G^{jg}\}\},$$
$$R_{139}^{(ij)(ab8)} = f^{8eg} f^{aeh} \{T^g, \{G^{ih}, G^{jb}\}\}, \qquad R_{140}^{(ij)(ab8)} = f^{beg} f^{8eh} \{T^a, \{G^{ig}, G^{jh}\}\},$$
$$R_{141}^{(ij)(ab8)} = f^{beg} f^{8eh} \{T^h, \{G^{ia}, G^{jg}\}\}, \qquad R_{142}^{(ij)(ab8)} = f^{beg} f^{8eh} \{T^g, \{G^{ih}, G^{ja}\}\},$$
$$R_{143}^{(ij)(ab8)} = f^{beg} f^{aeh} \{T^h, \{G^{i8}, G^{jg}\}\}, \qquad R_{144}^{(ij)(ab8)} = f^{8eg} f^{beh} \{T^h, \{G^{ia}, G^{jg}\}\},$$
$$R_{145}^{(ij)(ab8)} = f^{aeg} f^{8eh} \{T^h, \{G^{ib}, G^{jg}\}\}, \qquad R_{146}^{(ij)(ab8)} = d^{aeg} d^{beh} \{T^8, \{G^{ig}, G^{jh}\}\},$$
$$R_{147}^{(ij)(ab8)} = d^{aeg} d^{beh} \{T^h, \{G^{i8}, G^{jg}\}\}, \qquad R_{148}^{(ij)(ab8)} = d^{8eg} d^{aeh} \{T^b, \{G^{ig}, G^{jh}\}\},$$
$$R_{149}^{(ij)(ab8)} = d^{beg} d^{8eh} \{T^a, \{G^{ig}, G^{jh}\}\}, \qquad R_{150}^{(ij)(ab8)} = i d^{aeg} f^{beh} \{T^8, \{G^{ig}, G^{jh}\}\},$$
$$R_{151}^{(ij)(ab8)} = i d^{aeg} f^{beh} \{T^h, \{G^{i8}, G^{jg}\}\}, \qquad R_{152}^{(ij)(ab8)} = i d^{aeg} f^{beh} \{T^g, \{G^{ih}, G^{j8}\}\},$$
$$R_{153}^{(ij)(ab8)} = i d^{8eg} f^{aeh} \{T^b, \{G^{ig}, G^{jh}\}\}, \qquad R_{154}^{(ij)(ab8)} = i d^{8eg} f^{aeh} \{T^h, \{G^{ib}, G^{jg}\}\},$$
$$R_{155}^{(ij)(ab8)} = i d^{8eg} f^{aeh} \{T^g, \{G^{ih}, G^{jb}\}\}, \qquad R_{156}^{(ij)(ab8)} = i d^{beg} f^{8eh} \{T^a, \{G^{ig}, G^{jh}\}\},$$
$$R_{157}^{(ij)(ab8)} = i d^{beg} f^{8eh} \{T^h, \{G^{ia}, G^{jg}\}\}, \qquad R_{158}^{(ij)(ab8)} = i d^{beg} f^{8eh} \{T^g, \{G^{ih}, G^{ja}\}\},$$
$$R_{159}^{(ij)(ab8)} = i d^{beg} f^{aeh} \{T^h, \{G^{i8}, G^{jg}\}\}, \qquad R_{160}^{(ij)(ab8)} = i d^{beg} f^{aeh} \{T^g, \{G^{ih}, G^{j8}\}\},$$
$$R_{161}^{(ij)(ab8)} = i d^{8eg} f^{beh} \{T^h, \{G^{ia}, G^{jg}\}\}, \qquad R_{162}^{(ij)(ab8)} = i d^{8eg} f^{beh} \{T^g, \{G^{ih}, G^{ja}\}\},$$
$$R_{163}^{(ij)(ab8)} = i d^{aeg} f^{8eh} \{T^h, \{G^{ib}, G^{jg}\}\}, \qquad R_{164}^{(ij)(ab8)} = i d^{aeg} f^{8eh} \{T^g, \{G^{ih}, G^{jb}\}\},$$
$$R_{165}^{(ij)(ab8)} = i d^{aeg} f^{beh} \{T^h, \{G^{j8}, G^{ig}\}\}, \qquad R_{166}^{(ij)(ab8)} = i d^{aeg} f^{beh} \{T^g, \{G^{jh}, G^{i8}\}\},$$
$$R_{167}^{(ij)(ab8)} = i d^{8eg} f^{aeh} \{T^h, \{G^{jb}, G^{ig}\}\}, \qquad R_{168}^{(ij)(ab8)} = i d^{8eg} f^{aeh} \{T^g, \{G^{jh}, G^{ib}\}\},$$
$$R_{169}^{(ij)(ab8)} = i d^{beg} f^{8eh} \{T^h, \{G^{ja}, G^{ig}\}\}, \qquad R_{170}^{(ij)(ab8)} = i d^{beg} f^{8eh} \{T^g, \{G^{jh}, G^{ia}\}\}.$$

Table C.1: Non-trivial matrix elements of the 1-, 2-, and 3-body operators corresponding to the scattering amplitude of the process $p+\pi^+ \to p+\pi^+$. The entries correspond to $2\sqrt{3}k^ik'^j\langle[\mathcal{P}^{(m)}R_n^{(ij)}]^{(ab8)}\rangle$.

	$\mathbf{1}$	$\mathbf{8}$	$\mathbf{10}+\overline{\mathbf{10}}$	$\mathbf{27}$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{2}^{(ij)}]^{(ab8)}\rangle$	$2\mathbf{k}\cdot\mathbf{k}'$	0	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{4}^{(ij)}]^{(ab8)}\rangle$	$i(\mathbf{k}\times\mathbf{k}')^3$	0	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{5}^{(ij)}]^{(ab8)}\rangle$	0	$3\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{8}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{13}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{1}{6}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{14}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{5}{6}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{18}^{(ij)}]^{(ab8)}\rangle$	$\mathbf{k}\cdot\mathbf{k}'$	0	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{20}^{(ij)}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{21}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{3}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{26}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{27}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{30}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{33}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{40}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{1}{6}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{41}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{1}{6}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{42}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{5}{6}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{45}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{5}{6}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{48}^{(ij)}]^{(ab8)}\rangle$	0	$-3\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{51}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{9}{5}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{6}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{52}^{(ij)}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{5}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{9}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{53}^{(ij)}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{3}{5}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{54}^{(ij)}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{3}{5}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{55}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{9}{5}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{6}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{57}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{11}{10}i(\mathbf{k}\times\mathbf{k}')^3$	$-i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{2}{5}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{60}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{11}{10}i(\mathbf{k}\times\mathbf{k}')^3$	$i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{2}{5}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{63}^{(ij)}]^{(ab8)}\rangle$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{64}^{(ij)}]^{(ab8)}\rangle$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{30}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{65}^{(ij)}]^{(ab8)}\rangle$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{30}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{66}^{(ij)}]^{(ab8)}\rangle$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{67}^{(ij)}]^{(ab8)}\rangle$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{30}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{68}^{(ij)}]^{(ab8)}\rangle$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{30}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{69}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{41}{40}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{71}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{41}{40}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{73}^{(ij)}]^{(ab8)}\rangle$	$\frac{3}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{74}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{9}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{77}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{9}{5}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{6}{5}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{80}^{(ij)}]^{(ab8)}\rangle$	$\frac{3}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{5}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{9}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{81}^{(ij)}]^{(ab8)}\rangle$	$\frac{3}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{5}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{82}^{(ij)}]^{(ab8)}\rangle$	$\frac{3}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{5}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{83}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{11}{10}\mathbf{k}\cdot\mathbf{k}'$	$-\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{86}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{11}{10}\mathbf{k}\cdot\mathbf{k}'$	$-\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{89}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{11}{10}\mathbf{k}\cdot\mathbf{k}'$	$\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{92}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{11}{10}\mathbf{k}\cdot\mathbf{k}'$	$\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{95}^{(ij)}]^{(ab8)}\rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$

Table C.2: First continuation of above table.

	$\mathbf{1}$	$\mathbf{8}$	$\mathbf{10}+\overline{\mathbf{10}}$	$\mathbf{27}$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{96}^{(ij)}]^{(ab8)}\rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{97}^{(ij)}]^{(ab8)}\rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{98}^{(ij)}]^{(ab8)}\rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{99}^{(ij)}]^{(ab8)}\rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{100}^{(ij)}]^{(ab8)}\rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{101}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{3}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{107}^{(ij)}]^{(ab8)}\rangle$	0	$3\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{108}^{(ij)}]^{(ab8)}\rangle$	0	$3\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{109}^{(ij)}]^{(ab8)}\rangle$	0	$3\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{110}^{(ij)}]^{(ab8)}\rangle$	$3\mathbf{k}\cdot\mathbf{k}'$	$-\frac{39}{20}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{111}^{(ij)}]^{(ab8)}\rangle$	$3\mathbf{k}\cdot\mathbf{k}'$	$\frac{189}{20}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{9}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{112}^{(ij)}]^{(ab8)}\rangle$	$3\mathbf{k}\cdot\mathbf{k}'$	$-\frac{39}{20}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{113}^{(ij)}]^{(ab8)}\rangle$	0	$19\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{116}^{(ij)}]^{(ab8)}\rangle$	$\mathbf{k}\cdot\mathbf{k}'$	$-\frac{13}{20}\mathbf{k}\cdot\mathbf{k}'+\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{15}\mathbf{k}\cdot\mathbf{k}'-\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{117}^{(ij)}]^{(ab8)}\rangle$	$\mathbf{k}\cdot\mathbf{k}'$	$\frac{63}{20}\mathbf{k}\cdot\mathbf{k}'-\frac{5}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$-i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{5}\mathbf{k}\cdot\mathbf{k}'-\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{118}^{(ij)}]^{(ab8)}\rangle$	$\mathbf{k}\cdot\mathbf{k}'$	$-\frac{13}{20}\mathbf{k}\cdot\mathbf{k}'+\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{15}\mathbf{k}\cdot\mathbf{k}'-\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{119}^{(ij)}]^{(ab8)}\rangle$	$\mathbf{k}\cdot\mathbf{k}'$	$-\frac{13}{20}\mathbf{k}\cdot\mathbf{k}'-\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{15}\mathbf{k}\cdot\mathbf{k}'+\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{120}^{(ij)}]^{(ab8)}\rangle$	$\mathbf{k}\cdot\mathbf{k}'$	$\frac{63}{20}\mathbf{k}\cdot\mathbf{k}'+\frac{5}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{5}\mathbf{k}\cdot\mathbf{k}'+\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{121}^{(ij)}]^{(ab8)}\rangle$	$\mathbf{k}\cdot\mathbf{k}'$	$-\frac{13}{20}\mathbf{k}\cdot\mathbf{k}'-\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{15}\mathbf{k}\cdot\mathbf{k}'+\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{122}^{(ij)}]^{(ab8)}\rangle$	$\frac{3}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{13}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{9}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{123}^{(ij)}]^{(ab8)}\rangle$	$\frac{3}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{7}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{124}^{(ij)}]^{(ab8)}\rangle$	$\frac{3}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{13}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{9}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{125}^{(ij)}]^{(ab8)}\rangle$	$\frac{9}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{19}{20}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{13}{40}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{126}^{(ij)}]^{(ab8)}\rangle$	$\frac{9}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{13}{20}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{21}{40}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{127}^{(ij)}]^{(ab8)}\rangle$	$\frac{9}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{13}{20}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{21}{40}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{128}^{(ij)}]^{(ab8)}\rangle$	$-\frac{3}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{101}{60}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{13}{120}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{129}^{(ij)}]^{(ab8)}\rangle$	$-\frac{3}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{13}{60}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{7}{40}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{130}^{(ij)}]^{(ab8)}\rangle$	$-\frac{3}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{13}{60}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{7}{40}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{131}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{1}{20}i(\mathbf{k}\times\mathbf{k}')^3$	$i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{134}^{(ij)}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{329}{40}\mathbf{k}\cdot\mathbf{k}'-\frac{57}{40}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{9}{10}\mathbf{k}\cdot\mathbf{k}'-\frac{9}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{135}^{(ij)}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{107}{80}\mathbf{k}\cdot\mathbf{k}'-\frac{21}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{7}{30}\mathbf{k}\cdot\mathbf{k}'-\frac{1}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{136}^{(ij)}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{107}{80}\mathbf{k}\cdot\mathbf{k}'-\frac{21}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{7}{30}\mathbf{k}\cdot\mathbf{k}'-\frac{1}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{137}^{(ij)}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{49}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{138}^{(ij)}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{80}\mathbf{k}\cdot\mathbf{k}'+\frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'-\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{139}^{(ij)}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{80}\mathbf{k}\cdot\mathbf{k}'-\frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'+\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{140}^{(ij)}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{49}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{141}^{(ij)}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{80}\mathbf{k}\cdot\mathbf{k}'-\frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'+\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{142}^{(ij)}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{80}\mathbf{k}\cdot\mathbf{k}'+\frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'-\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{143}^{(ij)}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{107}{80}\mathbf{k}\cdot\mathbf{k}'+\frac{21}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{7}{30}\mathbf{k}\cdot\mathbf{k}'+\frac{1}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{144}^{(ij)}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{80}\mathbf{k}\cdot\mathbf{k}'-\frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'+\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{145}^{(ij)}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{80}\mathbf{k}\cdot\mathbf{k}'+\frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'-\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{146}^{(ij)}]^{(ab8)}\rangle$	$-\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{431}{120}\mathbf{k}\cdot\mathbf{k}'-\frac{71}{120}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{5}{6}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'-\frac{7}{60}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{147}^{(ij)}]^{(ab8)}\rangle$	$-\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{29}{80}\mathbf{k}\cdot\mathbf{k}'-\frac{37}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{7}{90}\mathbf{k}\cdot\mathbf{k}'+\frac{3}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{148}^{(ij)}]^{(ab8)}\rangle$	$-\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{120}\mathbf{k}\cdot\mathbf{k}'-\frac{1}{5}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'+\frac{1}{5}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{149}^{(ij)}]^{(ab8)}\rangle$	$-\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{120}\mathbf{k}\cdot\mathbf{k}'-\frac{1}{5}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'+\frac{1}{5}i(\mathbf{k}\times\mathbf{k}')^3$

Table C.3: Second continuation of above table.

$k^i k'^j \langle [\mathcal{P}^{(m)} R_{150}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{17}{24}\mathbf{k}\cdot\mathbf{k}' + \frac{21}{40} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{7}{12}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{3}\mathbf{k}\cdot\mathbf{k}' - \frac{27}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{151}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{77}{240}\mathbf{k}\cdot\mathbf{k}' + \frac{33}{80} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{15}\mathbf{k}\cdot\mathbf{k}' - \frac{3}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{152}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{29}{240}\mathbf{k}\cdot\mathbf{k}' + \frac{33}{80} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{3}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{153}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{33}{40} i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{19}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{154}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{63}{80}\mathbf{k}\cdot\mathbf{k}' - \frac{15}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{13}{48} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{155}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{63}{80}\mathbf{k}\cdot\mathbf{k}' - \frac{15}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{13}{48} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{156}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{9}{40} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{13}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{157}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{47}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{9}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' + \frac{21}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{158}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{47}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{9}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' + \frac{21}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{159}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{77}{240}\mathbf{k}\cdot\mathbf{k}' + \frac{33}{80} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{15}\mathbf{k}\cdot\mathbf{k}' - \frac{3}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{160}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{29}{240}\mathbf{k}\cdot\mathbf{k}' + \frac{33}{80} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{3}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{161}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{63}{80}\mathbf{k}\cdot\mathbf{k}' - \frac{15}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{13}{48} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{162}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{63}{80}\mathbf{k}\cdot\mathbf{k}' - \frac{15}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{13}{48} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{163}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{47}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{9}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' + \frac{21}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{164}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{47}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{9}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' + \frac{21}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{165}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{77}{240}\mathbf{k}\cdot\mathbf{k}' - \frac{33}{80} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{15}\mathbf{k}\cdot\mathbf{k}' + \frac{3}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{166}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{29}{240}\mathbf{k}\cdot\mathbf{k}' - \frac{33}{80} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' + \frac{3}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{167}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{63}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{15}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' + \frac{13}{48} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{168}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{63}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{15}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' + \frac{13}{48} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{169}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{47}{80}\mathbf{k}\cdot\mathbf{k}' - \frac{9}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{21}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{170}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{47}{80}\mathbf{k}\cdot\mathbf{k}' - \frac{9}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{21}{80} i(\mathbf{k}\times\mathbf{k}')^3$

Table C.4: Non-trivial matrix elements of the 1-, 2-, and 3-body operators corresponding to the scattering amplitude of the process $p + \pi^0 \to p + \pi^0$. The entries correspond to $\sqrt{3}k^i k'^j \langle [\mathcal{P}^{(m)} R_n^{(ij)}]^{(ab8)} \rangle$.

	$\mathbf{1}$	$\mathbf{8}$	$\mathbf{10}+\overline{\mathbf{10}}$	$\mathbf{27}$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{2}^{(ij)}]^{(ab8)} \rangle$	$\mathbf{k}\cdot\mathbf{k}'$	0	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{4}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{5}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{8}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{1}{4}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{13}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{1}{12}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{18}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{20}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	0	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{21}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{3}{4}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{26}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{1}{4}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{30}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{33}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{40}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{41}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{48}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{52}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{9}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{53}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{54}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{63}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{20}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{3}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{64}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{20}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{60}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{65}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{20}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{60}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{66}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{20}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{3}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{67}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{20}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{60}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{68}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{20}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{60}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{73}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}i(\mathbf{k}\times\mathbf{k}')^3$	0	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{74}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{9}{4}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{80}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{9}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{81}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{82}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{95}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{96}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{60}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{97}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{60}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{98}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{99}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{60}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{100}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{60}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{101}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{3}{4}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{107}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{108}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{109}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{110}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{39}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{111}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{189}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{9}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{112}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{39}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{113}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{19}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{116}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{13}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{117}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{63}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{118}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{13}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{119}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{13}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'$

Table C.5: First continuation of above table.

$k^i k'^j \langle [\mathcal{P}^{(m)} R_{120}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{63}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{121}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{13}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{122}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{13}{20}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{9}{40}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{123}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{20}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{7}{40}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{124}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{13}{20}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{9}{40}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{125}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{16}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{19}{40}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{13}{80}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{126}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{16}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{13}{40}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{21}{80}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{127}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{16}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{13}{40}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{21}{80}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{128}^{(ij)}]^{(ab8)} \rangle$	$-\frac{3}{16}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{101}{120}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{13}{240}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{129}^{(ij)}]^{(ab8)} \rangle$	$-\frac{3}{16}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{13}{120}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{7}{80}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{130}^{(ij)}]^{(ab8)} \rangle$	$-\frac{3}{16}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{13}{120}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{7}{80}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{134}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{329}{80}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{9}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{135}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{107}{160}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{7}{60}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{136}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{107}{160}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{7}{60}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{137}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{49}{80}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{138}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{160}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{139}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{160}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{140}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{49}{80}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{141}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{160}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{142}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{160}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{143}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{107}{160}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{7}{60}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{144}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{160}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{145}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{160}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{146}^{(ij)}]^{(ab8)} \rangle$	$-\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{431}{240}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{147}^{(ij)}]^{(ab8)} \rangle$	$-\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{29}{160}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{7}{180}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{148}^{(ij)}]^{(ab8)} \rangle$	$-\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{240}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{1}{60}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{149}^{(ij)}]^{(ab8)} \rangle$	$-\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{240}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{1}{60}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{150}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{21}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{27}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{151}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{33}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{152}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{33}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{153}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{33}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{19}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{154}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{15}{32}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{13}{96}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{155}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{15}{32}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{13}{96}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{156}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{9}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{13}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{157}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{9}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{21}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{158}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{9}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{21}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{159}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{33}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{160}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{33}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{161}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{15}{32}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{13}{96}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{162}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{15}{32}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{13}{96}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{163}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{9}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{21}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{164}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{9}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{21}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{165}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{33}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{3}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{166}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{33}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{3}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{167}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{15}{32}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{13}{96}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{168}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{15}{32}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{13}{96}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{169}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{9}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{21}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{170}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{9}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{21}{160}i(\mathbf{k}\times\mathbf{k}')^3$

Table C.6: Non-trivial matrix elements of the 1-, 2-, and 3-body operators corresponding to the scattering amplitude of the process $p + \pi^- \to n + \pi^0$. The entries correspond to $\sqrt{6}k^i k'^j \langle [\mathcal{P}^{(m)} R_n^{(ij)}]^{(ab8)} \rangle$.

	$\mathbf{1}$	$\mathbf{8}$	$\mathbf{10}+\overline{\mathbf{10}}$	$\mathbf{27}$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{14}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{5}{6} i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{27}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{1}{2} i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{42}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{5}{6} \mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{45}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{5}{6} \mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{51}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{9}{5} \mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{6}{5} \mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{55}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{9}{5} \mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{6}{5} \mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{57}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{11}{10} i(\mathbf{k}\times\mathbf{k}')^3$	$-i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{2}{5} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{60}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{11}{10} i(\mathbf{k}\times\mathbf{k}')^3$	$i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{2}{5} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{69}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{41}{40} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{12} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{10} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{71}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{41}{40} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{12} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{10} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{77}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{9}{5} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{6}{5} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{83}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{11}{10} \mathbf{k}\cdot\mathbf{k}'$	$-\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5} \mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{86}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{11}{10} \mathbf{k}\cdot\mathbf{k}'$	$-\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5} \mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{89}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{11}{10} \mathbf{k}\cdot\mathbf{k}'$	$\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5} \mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{92}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{11}{10} \mathbf{k}\cdot\mathbf{k}'$	$\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5} \mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{116}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{3}{10} i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{10} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{117}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{5}{4} i(\mathbf{k}\times\mathbf{k}')^3$	$-i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{2} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{118}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{3}{10} i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{10} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{119}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{3}{10} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{3}{10} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{120}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{5}{4} i(\mathbf{k}\times\mathbf{k}')^3$	$i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{2} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{121}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{3}{10} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{3}{10} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{131}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{1}{20} i(\mathbf{k}\times\mathbf{k}')^3$	$i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{10} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{134}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{57}{40} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{2} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{9}{20} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{135}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{21}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{1}{20} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{136}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{21}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{1}{20} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{138}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{93}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{1}{10} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{139}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{93}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{10} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{141}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{93}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{10} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{142}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{93}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{1}{10} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{143}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{21}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{20} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{144}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{93}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{10} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{145}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{93}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{1}{10} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{146}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{71}{120} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{5}{6} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{7}{60} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{147}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{37}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{3}{20} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{148}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{1}{5} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{5} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{149}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{1}{5} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{5} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{150}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{17}{24} \mathbf{k}\cdot\mathbf{k}'$	$-\frac{7}{12} \mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{3} \mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{151}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{77}{240} \mathbf{k}\cdot\mathbf{k}'$	$-\frac{1}{12} \mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{15} \mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{152}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{29}{240} \mathbf{k}\cdot\mathbf{k}'$	$-\frac{1}{12} \mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5} \mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{154}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{63}{80} \mathbf{k}\cdot\mathbf{k}'$	$-\frac{1}{3} \mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5} \mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{155}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{63}{80} \mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{3} \mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5} \mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{157}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{47}{80} \mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{2}{5} \mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{158}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{47}{80} \mathbf{k}\cdot\mathbf{k}'$	0	$\frac{2}{5} \mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{159}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{77}{240} \mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{12} \mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{15} \mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{160}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{29}{240} \mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{12} \mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5} \mathbf{k}\cdot\mathbf{k}'$

Table C.7: First continuation of above table.

$k^ik'^j\langle[\mathcal{P}^{(m)}R_{161}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{63}{80}\mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{162}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{63}{80}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{163}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{47}{80}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{164}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{47}{80}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{165}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{77}{240}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{15}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{166}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{29}{240}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{167}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{63}{80}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{168}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{63}{80}\mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{169}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{47}{80}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{170}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{47}{80}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$

Table C.8: Non-trivial matrix elements of the 1-, 2-, and 3-body operators corresponding to the scattering amplitude of the process $n + \pi^+ \to n + \pi^+$. The entries correspond to $2\sqrt{3}k^i k'^j \langle [\mathcal{P}^{(m)} R_n^{(ij)}]^{(ab8)} \rangle$.

	$\mathbf{1}$	$\mathbf{8}$	$\mathbf{10}+\overline{\mathbf{10}}$	$\mathbf{27}$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{2}^{(ij)}]^{(ab8)} \rangle$	$2\mathbf{k}\cdot\mathbf{k}'$	0	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{4}^{(ij)}]^{(ab8)} \rangle$	$i(\mathbf{k}\times\mathbf{k}')^3$	0	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{5}^{(ij)}]^{(ab8)} \rangle$	0	$3\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{8}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{13}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{1}{6}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{14}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{5}{6}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{18}^{(ij)}]^{(ab8)} \rangle$	$\mathbf{k}\cdot\mathbf{k}'$	0	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{20}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{21}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{3}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{26}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{27}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{30}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{33}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{40}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{1}{6}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{41}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{1}{6}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{42}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{5}{6}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{45}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{5}{6}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{48}^{(ij)}]^{(ab8)} \rangle$	0	$-3\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{51}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{9}{5}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{6}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{52}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{5}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{9}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{53}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{3}{5}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{54}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{3}{5}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{55}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{9}{5}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{6}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{57}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{11}{10}i(\mathbf{k}\times\mathbf{k}')^3$	$i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{2}{5}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{60}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{11}{10}i(\mathbf{k}\times\mathbf{k}')^3$	$-i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{2}{5}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{63}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{64}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{30}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{65}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{30}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{66}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{67}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{30}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{68}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{30}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{69}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{41}{40}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{71}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{41}{40}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{73}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{74}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{9}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{77}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{9}{5}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{6}{5}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{80}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{5}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{9}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{81}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{5}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{82}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{5}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{83}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{11}{10}\mathbf{k}\cdot\mathbf{k}'$	$\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{86}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{11}{10}\mathbf{k}\cdot\mathbf{k}'$	$\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{89}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{11}{10}\mathbf{k}\cdot\mathbf{k}'$	$-\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{92}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{11}{10}\mathbf{k}\cdot\mathbf{k}'$	$-\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{95}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$

Table C.9: First continuation of above table.

$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{96}]^{(ab8)} \rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{97}]^{(ab8)} \rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{98}]^{(ab8)} \rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{99}]^{(ab8)} \rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{100}]^{(ab8)} \rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{101}]^{(ab8)} \rangle$	0	$\frac{3}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{107}]^{(ab8)} \rangle$	0	$3\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{108}]^{(ab8)} \rangle$	0	$3\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{109}]^{(ab8)} \rangle$	0	$3\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{110}]^{(ab8)} \rangle$	$3\mathbf{k}\cdot\mathbf{k}'$	$-\frac{39}{20}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{111}]^{(ab8)} \rangle$	$3\mathbf{k}\cdot\mathbf{k}'$	$\frac{189}{20}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{9}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{112}]^{(ab8)} \rangle$	$3\mathbf{k}\cdot\mathbf{k}'$	$-\frac{39}{20}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{113}]^{(ab8)} \rangle$	0	$19\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{116}]^{(ab8)} \rangle$	$\mathbf{k}\cdot\mathbf{k}'$	$-\frac{13}{20}\mathbf{k}\cdot\mathbf{k}' - \frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{15}\mathbf{k}\cdot\mathbf{k}' + \frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{117}]^{(ab8)} \rangle$	$\mathbf{k}\cdot\mathbf{k}'$	$\frac{63}{20}\mathbf{k}\cdot\mathbf{k}' + \frac{5}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{5}\mathbf{k}\cdot\mathbf{k}' + \frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{118}]^{(ab8)} \rangle$	$\mathbf{k}\cdot\mathbf{k}'$	$-\frac{13}{20}\mathbf{k}\cdot\mathbf{k}' - \frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{15}\mathbf{k}\cdot\mathbf{k}' + \frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{119}]^{(ab8)} \rangle$	$\mathbf{k}\cdot\mathbf{k}'$	$-\frac{13}{20}\mathbf{k}\cdot\mathbf{k}' + \frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{15}\mathbf{k}\cdot\mathbf{k}' - \frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{120}]^{(ab8)} \rangle$	$\mathbf{k}\cdot\mathbf{k}'$	$\frac{63}{20}\mathbf{k}\cdot\mathbf{k}' - \frac{5}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$-i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{121}]^{(ab8)} \rangle$	$\mathbf{k}\cdot\mathbf{k}'$	$-\frac{13}{20}\mathbf{k}\cdot\mathbf{k}' + \frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{15}\mathbf{k}\cdot\mathbf{k}' - \frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{122}]^{(ab8)} \rangle$	$\frac{3}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{13}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{9}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{123}]^{(ab8)} \rangle$	$\frac{3}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{7}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{124}]^{(ab8)} \rangle$	$\frac{3}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{13}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{9}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{125}]^{(ab8)} \rangle$	$\frac{9}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{19}{20}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{13}{40}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{126}]^{(ab8)} \rangle$	$\frac{9}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{13}{20}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{21}{40}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{127}]^{(ab8)} \rangle$	$\frac{9}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{13}{20}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{21}{40}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{128}]^{(ab8)} \rangle$	$-\frac{3}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{101}{60}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{13}{120}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{129}]^{(ab8)} \rangle$	$-\frac{3}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{13}{60}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{7}{40}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{130}]^{(ab8)} \rangle$	$-\frac{3}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{13}{60}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{7}{40}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{131}]^{(ab8)} \rangle$	0	$-\frac{1}{20}i(\mathbf{k}\times\mathbf{k}')^3$	$-i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{134}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{329}{40}\mathbf{k}\cdot\mathbf{k}' + \frac{57}{40}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{9}{10}\mathbf{k}\cdot\mathbf{k}' + \frac{9}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{135}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{107}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{21}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{7}{30}\mathbf{k}\cdot\mathbf{k}' + \frac{1}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{136}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{107}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{21}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{7}{30}\mathbf{k}\cdot\mathbf{k}' + \frac{1}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{137}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{49}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{138}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{80}\mathbf{k}\cdot\mathbf{k}' - \frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}' + \frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{139}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}' - \frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{140}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{49}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{141}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}' - \frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{142}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{80}\mathbf{k}\cdot\mathbf{k}' - \frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}' + \frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{143}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{107}{80}\mathbf{k}\cdot\mathbf{k}' - \frac{21}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{7}{30}\mathbf{k}\cdot\mathbf{k}' - \frac{1}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{144}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}' - \frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{145}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{80}\mathbf{k}\cdot\mathbf{k}' - \frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}' + \frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{146}]^{(ab8)} \rangle$	$-\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{431}{120}\mathbf{k}\cdot\mathbf{k}' + \frac{71}{120}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{5}{6}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}' + \frac{7}{60}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{147}]^{(ab8)} \rangle$	$-\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{29}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{37}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{7}{90}\mathbf{k}\cdot\mathbf{k}' - \frac{3}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{148}]^{(ab8)} \rangle$	$-\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{120}\mathbf{k}\cdot\mathbf{k}' + \frac{1}{5}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{1}{30}\mathbf{k}\cdot\mathbf{k}' - \frac{1}{5}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R^{(ij)}_{149}]^{(ab8)} \rangle$	$-\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{120}\mathbf{k}\cdot\mathbf{k}' + \frac{1}{5}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{1}{30}\mathbf{k}\cdot\mathbf{k}' - \frac{1}{5}i(\mathbf{k}\times\mathbf{k}')^3$

Table C.10: Second continuation of above table.

$k^i k'^j \langle [\mathcal{P}^{(m)} R_{150}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{17}{24}\mathbf{k}\cdot\mathbf{k}' + \frac{21}{40} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{7}{12}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{3}\mathbf{k}\cdot\mathbf{k}' - \frac{27}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{151}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{77}{240}\mathbf{k}\cdot\mathbf{k}' + \frac{33}{80} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{15}\mathbf{k}\cdot\mathbf{k}' - \frac{3}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{152}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{29}{240}\mathbf{k}\cdot\mathbf{k}' + \frac{33}{80} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{3}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{153}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{33}{40} i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{19}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{154}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{63}{80}\mathbf{k}\cdot\mathbf{k}' - \frac{15}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{13}{48} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{155}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{63}{80}\mathbf{k}\cdot\mathbf{k}' - \frac{15}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{13}{48} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{156}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{9}{40} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{13}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{157}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{47}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{9}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' + \frac{21}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{158}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{47}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{9}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' + \frac{21}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{159}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{77}{240}\mathbf{k}\cdot\mathbf{k}' + \frac{33}{80} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{15}\mathbf{k}\cdot\mathbf{k}' - \frac{3}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{160}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{29}{240}\mathbf{k}\cdot\mathbf{k}' + \frac{33}{80} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{3}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{161}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{63}{80}\mathbf{k}\cdot\mathbf{k}' - \frac{15}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{13}{48} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{162}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{63}{80}\mathbf{k}\cdot\mathbf{k}' - \frac{15}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{13}{48} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{163}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{47}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{9}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' + \frac{21}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{164}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{47}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{9}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' + \frac{21}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{165}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{77}{240}\mathbf{k}\cdot\mathbf{k}' - \frac{33}{80} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{15}\mathbf{k}\cdot\mathbf{k}' + \frac{3}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{166}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{29}{240}\mathbf{k}\cdot\mathbf{k}' - \frac{33}{80} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' + \frac{3}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{167}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{63}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{15}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' + \frac{13}{48} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{168}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{63}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{15}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' + \frac{13}{48} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{169}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{47}{80}\mathbf{k}\cdot\mathbf{k}' - \frac{9}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{21}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{170}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{47}{80}\mathbf{k}\cdot\mathbf{k}' - \frac{9}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{21}{80} i(\mathbf{k}\times\mathbf{k}')^3$

Table C.11: Non-trivial matrix elements of the 1-, 2-, and 3-body operators corresponding to the scattering amplitude of the process $n + \pi^- \to n + \pi^-$. The entries correspond to $2\sqrt{3}k^i k'^j \langle [\mathcal{P}^{(m)} R_n^{(ij)}]^{(ab8)} \rangle$.

	$\mathbf{1}$	$\mathbf{8}$	$\mathbf{10}+\overline{\mathbf{10}}$	$\mathbf{27}$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{2}^{(ij)}]^{(ab8)} \rangle$	$2\mathbf{k}\cdot\mathbf{k}'$	0	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{4}^{(ij)}]^{(ab8)} \rangle$	$i(\mathbf{k}\times\mathbf{k}')^3$	0	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{5}^{(ij)}]^{(ab8)} \rangle$	0	$3\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{8}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{13}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{1}{6}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{14}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{5}{6}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{18}^{(ij)}]^{(ab8)} \rangle$	$\mathbf{k}\cdot\mathbf{k}'$	0	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{20}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{21}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{3}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{26}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{27}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{30}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{33}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{40}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{1}{6}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{41}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{1}{6}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{42}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{5}{6}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{45}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{5}{6}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{48}^{(ij)}]^{(ab8)} \rangle$	0	$-3\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{51}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{9}{5}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{6}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{52}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{5}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{9}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{53}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{3}{5}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{54}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{3}{5}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{55}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{9}{5}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{6}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{57}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{11}{10}i(\mathbf{k}\times\mathbf{k}')^3$	$-i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{2}{5}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{60}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{11}{10}i(\mathbf{k}\times\mathbf{k}')^3$	$i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{2}{5}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{63}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{64}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{30}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{65}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{30}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{66}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{67}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{30}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{68}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{30}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{69}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{41}{40}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{71}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{41}{40}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{73}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{74}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{9}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{77}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{9}{5}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{6}{5}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{80}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{5}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{9}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{81}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{5}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{82}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{5}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{83}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{11}{10}\mathbf{k}\cdot\mathbf{k}'$	$-\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{86}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{11}{10}\mathbf{k}\cdot\mathbf{k}'$	$-\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{89}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{11}{10}\mathbf{k}\cdot\mathbf{k}'$	$\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{92}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{11}{10}\mathbf{k}\cdot\mathbf{k}'$	$\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{95}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$

Table C.12: First continuation of above table.

$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{96}]^{(ab8)}\rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{97}]^{(ab8)}\rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{98}]^{(ab8)}\rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{99}]^{(ab8)}\rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{100}]^{(ab8)}\rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{101}]^{(ab8)}\rangle$	0	$\frac{3}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{107}]^{(ab8)}\rangle$	0	$3\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{108}]^{(ab8)}\rangle$	0	$3\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{109}]^{(ab8)}\rangle$	0	$3\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{110}]^{(ab8)}\rangle$	$3\mathbf{k}\cdot\mathbf{k}'$	$-\frac{39}{20}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{111}]^{(ab8)}\rangle$	$3\mathbf{k}\cdot\mathbf{k}'$	$\frac{189}{20}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{9}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{112}]^{(ab8)}\rangle$	$3\mathbf{k}\cdot\mathbf{k}'$	$-\frac{39}{20}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{113}]^{(ab8)}\rangle$	0	$19\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{116}]^{(ab8)}\rangle$	$\mathbf{k}\cdot\mathbf{k}'$	$-\frac{13}{20}\mathbf{k}\cdot\mathbf{k}'+\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{15}\mathbf{k}\cdot\mathbf{k}'-\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{117}]^{(ab8)}\rangle$	$\mathbf{k}\cdot\mathbf{k}'$	$\frac{63}{20}\mathbf{k}\cdot\mathbf{k}'-\frac{5}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$-i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{5}\mathbf{k}\cdot\mathbf{k}'-\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{118}]^{(ab8)}\rangle$	$\mathbf{k}\cdot\mathbf{k}'$	$-\frac{13}{20}\mathbf{k}\cdot\mathbf{k}'+\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{15}\mathbf{k}\cdot\mathbf{k}'-\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{119}]^{(ab8)}\rangle$	$\mathbf{k}\cdot\mathbf{k}'$	$-\frac{13}{20}\mathbf{k}\cdot\mathbf{k}'-\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{15}\mathbf{k}\cdot\mathbf{k}'+\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{120}]^{(ab8)}\rangle$	$\mathbf{k}\cdot\mathbf{k}'$	$\frac{63}{20}\mathbf{k}\cdot\mathbf{k}'+\frac{5}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{5}\mathbf{k}\cdot\mathbf{k}'+\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{121}]^{(ab8)}\rangle$	$\mathbf{k}\cdot\mathbf{k}'$	$-\frac{13}{20}\mathbf{k}\cdot\mathbf{k}'-\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{15}\mathbf{k}\cdot\mathbf{k}'+\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{122}]^{(ab8)}\rangle$	$\frac{3}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{13}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{9}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{123}]^{(ab8)}\rangle$	$\frac{3}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{7}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{124}]^{(ab8)}\rangle$	$\frac{3}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{13}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{9}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{125}]^{(ab8)}\rangle$	$\frac{9}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{19}{20}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{13}{40}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{126}]^{(ab8)}\rangle$	$\frac{9}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{13}{20}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{21}{40}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{127}]^{(ab8)}\rangle$	$\frac{9}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{13}{20}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{21}{40}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{128}]^{(ab8)}\rangle$	$-\frac{3}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{101}{60}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{13}{120}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{129}]^{(ab8)}\rangle$	$-\frac{3}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{13}{60}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{7}{40}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{130}]^{(ab8)}\rangle$	$-\frac{3}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{13}{60}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{7}{40}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{131}]^{(ab8)}\rangle$	0	$\frac{1}{20}i(\mathbf{k}\times\mathbf{k}')^3$	$i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{134}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{329}{40}\mathbf{k}\cdot\mathbf{k}'-\frac{57}{40}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{9}{10}\mathbf{k}\cdot\mathbf{k}'-\frac{9}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{135}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{107}{80}\mathbf{k}\cdot\mathbf{k}'-\frac{21}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{7}{30}\mathbf{k}\cdot\mathbf{k}'-\frac{1}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{136}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{107}{80}\mathbf{k}\cdot\mathbf{k}'-\frac{21}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{7}{30}\mathbf{k}\cdot\mathbf{k}'-\frac{1}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{137}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{49}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{138}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{80}\mathbf{k}\cdot\mathbf{k}'+\frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'-\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{139}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{80}\mathbf{k}\cdot\mathbf{k}'-\frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'+\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{140}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{49}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{141}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{80}\mathbf{k}\cdot\mathbf{k}'-\frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'+\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{142}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{80}\mathbf{k}\cdot\mathbf{k}'+\frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'-\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{143}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{107}{80}\mathbf{k}\cdot\mathbf{k}'+\frac{21}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{7}{30}\mathbf{k}\cdot\mathbf{k}'+\frac{1}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{144}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{80}\mathbf{k}\cdot\mathbf{k}'-\frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'+\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{145}]^{(ab8)}\rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{80}\mathbf{k}\cdot\mathbf{k}'+\frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'-\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{146}]^{(ab8)}\rangle$	$-\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{431}{120}\mathbf{k}\cdot\mathbf{k}'-\frac{71}{120}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{5}{6}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'-\frac{7}{60}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{147}]^{(ab8)}\rangle$	$-\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{29}{80}\mathbf{k}\cdot\mathbf{k}'-\frac{37}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{7}{90}\mathbf{k}\cdot\mathbf{k}'+\frac{3}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{148}]^{(ab8)}\rangle$	$-\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{120}\mathbf{k}\cdot\mathbf{k}'-\frac{1}{5}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'+\frac{1}{5}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{149}]^{(ab8)}\rangle$	$-\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{120}\mathbf{k}\cdot\mathbf{k}'-\frac{1}{5}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'+\frac{1}{5}i(\mathbf{k}\times\mathbf{k}')^3$

Table C.13: Second continuation of above table.

$k^i k'^j \langle [\mathcal{P}^{(m)} R_{150}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{17}{24}\mathbf{k}\cdot\mathbf{k}' + \frac{21}{40} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{7}{12}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{3}\mathbf{k}\cdot\mathbf{k}' - \frac{27}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{151}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{77}{240}\mathbf{k}\cdot\mathbf{k}' + \frac{33}{80} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{15}\mathbf{k}\cdot\mathbf{k}' - \frac{3}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{152}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{29}{240}\mathbf{k}\cdot\mathbf{k}' + \frac{33}{80} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{3}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{153}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{33}{40} i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{19}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{154}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{63}{80}\mathbf{k}\cdot\mathbf{k}' - \frac{15}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{13}{48} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{155}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{63}{80}\mathbf{k}\cdot\mathbf{k}' - \frac{15}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{13}{48} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{156}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{9}{40} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{13}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{157}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{47}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{9}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' + \frac{21}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{158}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{47}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{9}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' + \frac{21}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{159}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{77}{240}\mathbf{k}\cdot\mathbf{k}' + \frac{33}{80} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{15}\mathbf{k}\cdot\mathbf{k}' - \frac{3}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{160}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{29}{240}\mathbf{k}\cdot\mathbf{k}' + \frac{33}{80} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{3}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{161}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{63}{80}\mathbf{k}\cdot\mathbf{k}' - \frac{15}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{13}{48} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{162}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{63}{80}\mathbf{k}\cdot\mathbf{k}' - \frac{15}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{13}{48} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{163}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{47}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{9}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' + \frac{21}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{164}^{(ij)}]^{(ab8)} \rangle$	$-\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{47}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{9}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' + \frac{21}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{165}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{77}{240}\mathbf{k}\cdot\mathbf{k}' - \frac{33}{80} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{15}\mathbf{k}\cdot\mathbf{k}' + \frac{3}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{166}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{29}{240}\mathbf{k}\cdot\mathbf{k}' - \frac{33}{80} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' + \frac{3}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{167}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{63}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{15}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' + \frac{13}{48} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{168}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{63}{80}\mathbf{k}\cdot\mathbf{k}' + \frac{15}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' + \frac{13}{48} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{169}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{47}{80}\mathbf{k}\cdot\mathbf{k}' - \frac{9}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{21}{80} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{170}^{(ij)}]^{(ab8)} \rangle$	$\frac{9}{16} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{47}{80}\mathbf{k}\cdot\mathbf{k}' - \frac{9}{80} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}' - \frac{21}{80} i(\mathbf{k}\times\mathbf{k}')^3$

Table C.14: Non-trivial matrix elements of the 1-, 2-, and 3-body operators corresponding to the scattering amplitude of the process $n + \pi^0 \to n + \pi^0$. The entries correspond to $\sqrt{3} k^i k'^j \langle [\mathcal{P}^{(m)} R_n^{(ij)}]^{(ab8)} \rangle$.

	$\mathbf{1}$	$\mathbf{8}$	$\mathbf{10}+\overline{\mathbf{10}}$	$\mathbf{27}$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{2}^{(ij)}]^{(ab8)} \rangle$	$\mathbf{k}\cdot\mathbf{k}'$	0	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{4}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2} i(\mathbf{k}\times\mathbf{k}')^3$	0	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{5}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{8}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{1}{4} i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{13}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{1}{12} i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{18}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{20}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	0	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{21}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{3}{4} i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{26}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{1}{4} i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{30}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{33}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{40}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{41}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{48}^{(ij)}]^{(ab8)} \rangle$	0	$-\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{52}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{9}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{53}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{54}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{63}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{20} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{3}{20} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{64}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{20} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{60} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{65}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{20} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{60} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{66}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{20} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{3}{20} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{67}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{20} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{60} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{68}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{20} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{60} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{73}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4} i(\mathbf{k}\times\mathbf{k}')^3$	0	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{74}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{9}{4}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{80}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4} i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{3}{10} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{9}{20} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{81}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{10} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{20} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{82}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{4} i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{10} i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{20} i(\mathbf{k}\times\mathbf{k}')^3$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{95}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{96}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{60}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{97}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{60}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{98}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{99}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{60}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{100}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{60}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{101}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{3}{4} i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{107}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{108}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{109}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{110}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{39}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{111}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{189}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{9}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{112}^{(ij)}]^{(ab8)} \rangle$	$\frac{3}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{39}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{113}^{(ij)}]^{(ab8)} \rangle$	0	$\frac{19}{2}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{116}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{13}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{117}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{63}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{118}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{13}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'$
$k^i k'^j \langle [\mathcal{P}^{(m)} R_{119}^{(ij)}]^{(ab8)} \rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{13}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'$

Table C.15: First continuation of above table.

$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{120}]^{(ab8)}\rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$\frac{63}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{3}{10}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{121}]^{(ab8)}\rangle$	$\frac{1}{2}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{13}{40}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{30}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{122}]^{(ab8)}\rangle$	$\frac{3}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{13}{20}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{9}{40}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{123}]^{(ab8)}\rangle$	$\frac{3}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{20}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{7}{40}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{124}]^{(ab8)}\rangle$	$\frac{3}{8}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{13}{20}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{9}{40}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{125}]^{(ab8)}\rangle$	$\frac{9}{16}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{19}{40}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{13}{80}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{126}]^{(ab8)}\rangle$	$\frac{9}{16}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{13}{40}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{21}{80}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{127}]^{(ab8)}\rangle$	$\frac{9}{16}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{13}{40}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{21}{80}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{128}]^{(ab8)}\rangle$	$-\frac{3}{16}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{101}{120}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{13}{240}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{129}]^{(ab8)}\rangle$	$-\frac{3}{16}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{13}{120}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{7}{80}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{130}]^{(ab8)}\rangle$	$-\frac{3}{16}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{13}{120}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{7}{80}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{134}]^{(ab8)}\rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{329}{80}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{9}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{135}]^{(ab8)}\rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{107}{160}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{7}{60}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{136}]^{(ab8)}\rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{107}{160}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{7}{60}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{137}]^{(ab8)}\rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{49}{80}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{138}]^{(ab8)}\rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{160}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{139}]^{(ab8)}\rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{160}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{140}]^{(ab8)}\rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{49}{80}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{1}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{141}]^{(ab8)}\rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{160}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{142}]^{(ab8)}\rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{160}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{143}]^{(ab8)}\rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{107}{160}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{7}{60}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{144}]^{(ab8)}\rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{160}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{145}]^{(ab8)}\rangle$	$\frac{3}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{160}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{146}]^{(ab8)}\rangle$	$-\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{431}{240}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{3}{20}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{147}]^{(ab8)}\rangle$	$-\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{29}{160}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{7}{180}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{148}]^{(ab8)}\rangle$	$-\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{240}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{1}{60}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{149}]^{(ab8)}\rangle$	$-\frac{1}{4}\mathbf{k}\cdot\mathbf{k}'$	$\frac{49}{240}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{1}{60}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{150}]^{(ab8)}\rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{21}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{27}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{151}]^{(ab8)}\rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{33}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{152}]^{(ab8)}\rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{33}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{153}]^{(ab8)}\rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{33}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{19}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{154}]^{(ab8)}\rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{15}{32}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{13}{96}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{155}]^{(ab8)}\rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{15}{32}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{13}{96}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{156}]^{(ab8)}\rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{9}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{13}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{157}]^{(ab8)}\rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{9}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{21}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{158}]^{(ab8)}\rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{9}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{21}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{159}]^{(ab8)}\rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{33}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{160}]^{(ab8)}\rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{33}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{161}]^{(ab8)}\rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{15}{32}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{13}{96}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{162}]^{(ab8)}\rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{15}{32}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{13}{96}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{163}]^{(ab8)}\rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{9}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{21}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{164}]^{(ab8)}\rangle$	$-\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{9}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{21}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{165}]^{(ab8)}\rangle$	$\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{33}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{3}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{166}]^{(ab8)}\rangle$	$\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{33}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{3}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{167}]^{(ab8)}\rangle$	$\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{15}{32}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{13}{96}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{168}]^{(ab8)}\rangle$	$\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{15}{32}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{13}{96}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{169}]^{(ab8)}\rangle$	$\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{9}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{21}{160}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R^{(ij)}_{170}]^{(ab8)}\rangle$	$\frac{9}{32}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{9}{160}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{21}{160}i(\mathbf{k}\times\mathbf{k}')^3$

Table C.16: Non-trivial matrix elements of the 1-, 2-, and 3-body operators corresponding to the scattering amplitude of the process $n + \pi^+ \to p + \pi^0$. The entries correspond to $\sqrt{6}k^i k'^j \langle [\mathcal{P}^{(m)} R_n^{(ij)}]^{(ab8)} \rangle$.

	1	**8**	$\mathbf{10}+\overline{\mathbf{10}}$	**27**
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{14}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{5}{6}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{27}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{42}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{5}{6}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{45}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{5}{6}\mathbf{k}\cdot\mathbf{k}'$	0	0
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{51}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{9}{5}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{6}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{55}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{9}{5}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{6}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{57}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{11}{10}i(\mathbf{k}\times\mathbf{k}')^3$	$-i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{2}{5}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{60}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{11}{10}i(\mathbf{k}\times\mathbf{k}')^3$	$i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{2}{5}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{69}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{41}{40}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{71}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{41}{40}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{12}i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{77}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{9}{5}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{6}{5}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{83}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{11}{10}\mathbf{k}\cdot\mathbf{k}'$	$-\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{86}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{11}{10}\mathbf{k}\cdot\mathbf{k}'$	$-\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{89}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{11}{10}\mathbf{k}\cdot\mathbf{k}'$	$\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{92}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{11}{10}\mathbf{k}\cdot\mathbf{k}'$	$\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{116}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{117}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{5}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$-i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{118}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{119}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{120}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{5}{4}i(\mathbf{k}\times\mathbf{k}')^3$	$i(\mathbf{k}\times\mathbf{k}')^3$	$\frac{1}{2}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{121}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{131}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{1}{20}i(\mathbf{k}\times\mathbf{k}')^3$	$i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{134}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{57}{40}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{3}{2}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{9}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{135}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{21}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{1}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{136}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{21}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{1}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{138}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{139}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{141}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{142}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{143}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{21}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{144}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{145}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{93}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$-\frac{1}{10}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{146}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{71}{120}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{5}{6}i(\mathbf{k}\times\mathbf{k}')^3$	$-\frac{7}{60}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{147}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{37}{80}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{3}{20}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{148}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{1}{5}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{5}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{149}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{1}{5}i(\mathbf{k}\times\mathbf{k}')^3$	0	$\frac{1}{5}i(\mathbf{k}\times\mathbf{k}')^3$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{150}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{17}{24}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{7}{12}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{3}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{151}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{77}{240}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{15}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{152}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{29}{240}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{154}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{63}{80}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{155}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{63}{80}\mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{157}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{47}{80}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{158}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{47}{80}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{159}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{77}{240}\mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{15}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{160}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{29}{240}\mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$

Table C.17: First continuation of above table.

$k^ik'^j\langle[\mathcal{P}^{(m)}R_{161}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{63}{80}\mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{162}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{63}{80}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{163}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{47}{80}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{164}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{47}{80}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{165}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{77}{240}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{15}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{166}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{29}{240}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{1}{12}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{167}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{63}{80}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{168}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{63}{80}\mathbf{k}\cdot\mathbf{k}'$	$\frac{1}{3}\mathbf{k}\cdot\mathbf{k}'$	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{169}^{(ij)}]^{(ab8)}\rangle$	0	$\frac{47}{80}\mathbf{k}\cdot\mathbf{k}'$	0	$-\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$
$k^ik'^j\langle[\mathcal{P}^{(m)}R_{170}^{(ij)}]^{(ab8)}\rangle$	0	$-\frac{47}{80}\mathbf{k}\cdot\mathbf{k}'$	0	$\frac{2}{5}\mathbf{k}\cdot\mathbf{k}'$

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